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Acoustic Phenomena Associated with the Firing of a Gun

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THE sounds that arise directly or indirectly from the firing of a gun are of practical importance in sound ranging, the purpose of which is to furnish a map reference of an enemy gun emplacement from acoustic data supplied by the gun itself. The technic of the process would be considerably simplified if the firing of a single round from a gun resulted in one sound only—the report of the gun. Actually, three distinct sounds are generated, any or all of which may be audible to an appropriately placed observer. These sounds, in the order of their production, are:

(1) *The sound resulting from the sudden irruption into the atmosphere of the highly compressed gases propelling the projectile along the barrel as the projectile itself quits the muzzle.* This sound, the report proper, is analogous to that produced by the expulsion of the cork from a champagne bottle—the "bottle" in this case being the cylindrical barrel of the gun, and the "cork" the projectile. Regarding the barrel as an ordinary open-stopped pipe, the length of which is approximately one-quarter of the wavelength of sound in air, one can obtain an estimate of the fundamental frequency of this muzzle wave. This frequency is usually low, and the aerial vibration associated with it heavily damped. In view of the drastic nature of the disturbance originating the report, the initial amplitude of the vibratory motion is large, and it is for this reason that the velocity of travel of the muzzle wave is abnormally high in the immediate vicinity of the gun. However, at distances a few wavelengths away from the gun, the velocity of the muzzle wave is found to have reduced to that appropriate to a sound wave of small amplitude under like conditions.

(2) *The sound arising from the passage of the projectile through the air and for which the frictional resistance it encounters as it spins about its axis and travels along its trajectory is primarily responsible.* Unlike the muzzle wave, which originates from a definite point, these projectile waves originate from the various points successively occupied by the projectile in the course of its flight through the air. Such sounds, comparatively high in pitch, would appear to be attributable to some periodicity in structure in the aerial turbulence generated in the wake of the projectile.

When the velocity of the projectile is less than that of sound in air, the first audible evidence of the passage of the projectile through the air consists of a high-pitched whistle reminiscent of the swish produced by the sudden sweep of a flexible switch through the air. A marked drop in pitch due to the Doppler effect occurs in respect of the projectile waves that originate from points along the arc of the trajectory extending on either side of the vertical through the observer.

When the speed of the projectile exceeds that of sound in air, a much more abrupt and intense sound, resembling the crack of a whip, is heard. This arises from the reception of the shock, or

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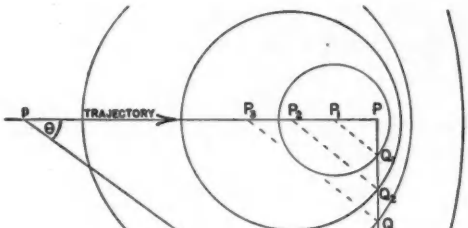

FIG. 1. Projectile-wave fronts. Horizontal trajectory; $v<c$.

ballistic, wave which, as will appear later, is confined to a conical layer of quite small thickness (of the order of 1 cm) within which the air is highly compressed. Its passage over the observer delivers a sudden impulse to his ear. The subjective effect of such a shock excitation, completed in less than 10^{-3} sec, is more correctly classified as a noise than a note, since it is impossible to assign to it any definite frequency. This is, in fact, the first sound that is heard, and, being often more pronounced than the report which is received later, is likely to be mistaken for the latter by an inexperienced observer, who concludes that the gun is being fired from a point much closer to him than it actually is.

The shock wave is followed by a momentary silence. Assuming, however, that it is legitimate to regard the projectile as constituting a permanent source of sound of definite frequency, the interval of silence will be succeeded by a sequence of sounds correctly classified as notes despite the fact that their frequencies are rendered variable through the agency of the Doppler effect. These sounds, as we shall see, may be either single or double. In the latter case the two component notes generally differ from each other in frequency, and arise from the simultaneous reception of projectile waves from the neighborhood of pairs of corresponding points on the trajectory on either side of the point located on the ascending arc of the trajectory from the immediate vicinity of which the shock wave itself originates.

(3) *The sound of the impact of the projectile originating from another separate and distinct point—presumably the target at which the projectile is fired.* The projectile is often designed to burst upon impact, and the resulting explosion wave resembles the muzzle wave, though it is generally even more heavily damped. This difference is mainly attributable to the choice of a propelling charge which, in comparison with the bursting charge, is relatively slow burning.

It will be assumed in what follows that all three types of wave travel through air with a speed equal to that of sound under the prevailing atmospheric conditions.

Velocity of Projectile Less Than That of Sound

Mention has been made of the fact that the acoustic phenomena resulting from the firing of a gun differ radically, according as v, the velocity of the projectile, is less or greater than c, the velocity of sound in air.

Suppose, for simplicity, the trajectory to be a horizontal straight line along which the projectile is traveling with uniform velocity v, less than c. Since the curvature of a trajectory is comparatively small (except at the summit in antiaircraft fire), this assumption will be legitimate, at least as a first approximation.

Then, as shown in Fig. 1, at the end of the time interval t during which the projectile travels from a point p to P, the sound energy generated during its passage through p will be localized over a spherical wave front of radius pO, where $pO[=ct]$ is greater than $pP[=vt]$. Let $P_1, P_2, \cdots$ represent the points along Pp occupied by the projectile 1 sec, 2 sec, $\cdots$ prior to its arrival at P. Then spheres described with centers P_1, P_2, $\cdots$ and with radii $P_1Q_1[=c]$, $P_2Q_2[=2c]$, $\cdots$ will represent the wave fronts over which the sound energy emitted as the projectile passes in turn through $\cdots, P_2, P_1$ is located at the instant the projectile itself reaches P. As Fig. 1 is drawn, the observer at O will just be receiving the projectile wave originating from p as the projectile itself is passing through P vertically over him. Since the respective members of this family of wave fronts nowhere intersect, the projectile waves originating from the positions successively occupied by the projectile pursuing its horizontal trajectory will arrive at the observer in the order of their production.

Now the projectile traveling with constant velocity v along its straight-line path will constitute a source of sound of period τ and frequency ν. The observer receiving the resulting sound waves will form his own independent

estimate of their frequency ν', and this, on account of the motion of the source, will differ, as a rule, from ν. The relation between ν' and ν can be derived by reference to Fig. 2(*a*), which is drawn for a value of θ (see Fig. 1) between O and $\frac{1}{2}\pi$. Suppose the projectile passing through P at a particular instant to be emitting, say, a compression crest. Suppose further that after an interval τ the projectile reaches P', where $PP'=v\tau=\lambda v/c$, and when it will be in process of emitting the succeeding compression crest. Then at the instant the compression crest is starting out from P', the previous compression crest will be localized over the surface of a sphere of center P and radius PS, where PS equals the wavelength λ of the sound as emitted by the source. Let O be the point occupied by the observer. Then, if with center O and radius OP' an arc is drawn to cut PO in Q, $P'O$ and QO will be acoustically equivalent paths in the sense that if a hypothetical compression crest were started from Q simultaneously with the actual compression crest from P', these would arrive simultaneously at O. Also, since the projectile is moving more slowly than sound, the compression crest from P will arrive at O ahead of that from P' or from Q which, as we have just seen, may be substituted in place of it.

The two consecutive compression crests propagated towards O along PO will be separated by the distance SQ, which will accordingly represent the wavelength λ' of the sound as received by the observer. But $SQ=PS-PQ$ and angle $POP'[\Delta\theta]$, being sufficiently small, we shall be justified in writing[1]

$$PQ=PP'\cos\theta=\lambda(v/c)\cos\theta.$$

Hence

$$\lambda'=\lambda\left(1-\frac{v}{c}\cos\theta\right)$$

or, since $\lambda/\lambda'=\nu'/\nu$,

$$\nu'=\frac{\nu}{1-(v/c)\cos\theta}.$$

When $\theta=0$,

$$\lambda_0'=S_0P'=PS_0-PP'=\lambda\left(1-\frac{v}{c}\right)$$

[1] The unabbreviated expression is

$$PQ=PP'\cos\theta(1-\tan\theta\tan\tfrac{1}{2}\Delta\theta),$$

where $POP'=\Delta\theta$, and the physical assumption implied is that OP is large compared to λ.

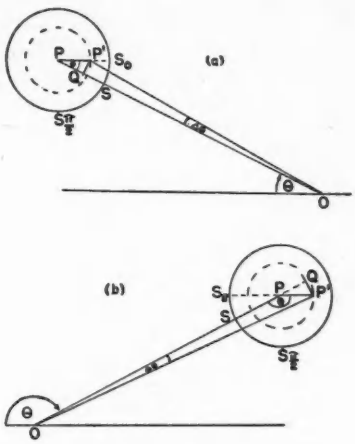

FIG. 2. Doppler effect; $v<c$.

and

$$\nu_0'=\frac{\nu}{1-(v/c)},$$

which agrees with the result obtained by direct substitution in the preceding formula. Increase of θ brought about by motion of the projectile towards the right or by an equivalent horizontal displacement of the observer towards the left will entail an increase in SQ, or λ', and a corresponding reduction in ν'. When $\theta=\frac{1}{2}\pi$, Q will become practically coincident with P so that $\lambda_{\pi/2}'=S_{\pi/2}P=\lambda$ and $\nu_{\pi/2}'=\nu$, again agreeing with the result of direct substitution in the original formula.

A precisely similar construction, indicated in Fig. 2(*b*), can be applied when θ lies between $\frac{1}{2}\pi$ and π, that is, when the projectile is receding from the observer. The arc of center O and radius OP' now intersects OP produced in Q, so that $SQ=PS+PQ$; that is,

$$\lambda'=\lambda\left[1+\frac{v}{c}\cos(\pi-\theta)\right],$$

or

$$\nu'=\frac{\nu}{1-(v/c)\cos\theta}$$

as before.

When $\theta=\pi$,

$$\lambda_\pi'=S_\pi P'=PS_\pi+PP'=\lambda(1+v/c),$$

and

$$\nu_\pi'=\frac{\nu}{1+(v/c)}.$$

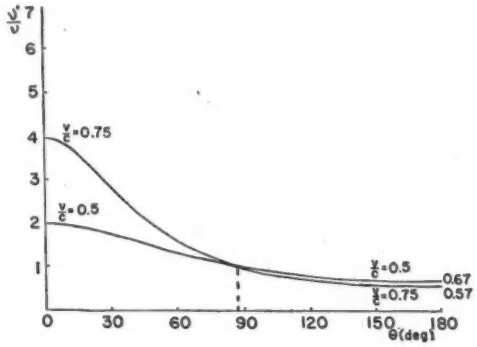

FIG. 3. Ratio of observed to actual frequency of projectile wave; $v<c$.

The manner in which ν'/ν, the ratio of the observed to the actual whistle frequency, varies with θ, the bearing of the point on the trajectory from which the corresponding projectile wave originates, is exhibited in the two curves of Fig. 3, corresponding, respectively, to values 0.5 and 0.75 of v/c. When $\theta=0$, the whistle as heard by the observer immediately succeeding the report will be an octave above the true pitch of the projectile wave when $v/c=0.5$, and two octaves above its true pitch when $v/c=0.75$.

When $\theta=\pi$, the whistle as heard by the observer immediately preceding the impact will be a fifth below the true pitch of the projectile wave when $v/c=0.5$, and slightly more than an augmented sixth below its true pitch when $v/c=0.75$.

The transition between the upper and lower limiting frequencies becomes progressively more abrupt as the observer approaches the trajectory.

The rate of change of ν' with θ, given by

$$\frac{d\nu'}{d\theta}=-\nu\frac{(v/c)\sin\theta}{[1-(v/c)\cos\theta]^2},$$

assumes its maximum value, $-\nu v/c$, when $\theta=\frac{1}{2}\pi$. Since this rate of change is zero for $\theta=0$ and for $\theta=\pi$, the approach to both limiting frequency values will be asymptotic.

Imagine now the restrictions previously imposed to be removed and suppose, as indicated in Fig. 4, the projectile leaving the gun at P_0 at the instant $t=0$ to pursue its curvilinear trajectory from P_0 to P_ω with a velocity v which, though varying from point to point, is yet everywhere less than c. Then an observer stationed at O and assumed located in the plane[2] of the trajectory will receive:

(i) The muzzle wave; and this, having traversed the distance P_0O with velocity c, will reach O after the interval $T_0=P_0O/c$.

[2] The actual trajectory will not normally be a plane curve owing to the drift resulting from the spin of the projectile.

(ii) The projectile waves emitted as the projectile passes in succession through consecutive points along its trajectory. If, for example, the projectile takes a time t to move along the arc from P_0 to a point such as P, then the projectile wave, starting out from P and traveling from P to O with velocity c, will reach O after an interval $T=t+(PO/c)$. Now it is obvious from the geometry of Fig. 4 that $P_0P+PO>P_0O$. Further, since ct, representing the distance a sound wave would travel in the time t, is greater than P_0P, the distance actually covered by the projectile in this same time (moving as it does throughout more slowly than sound), it follows, *a fortiori*, that $ct+PO>P_0O$, or $t+(PO/c)>P_0O/c$, so that $T>T_0$, which implies that the projectile wave reaches the observer later than the muzzle wave.

(iii) The impact wave initiated when the projectile, having occupied an interval t_ω in traversing its entire trajectory from P_0 to P_ω, hits its target. Since $P_\omega O$ will represent the distance the corresponding wave has to travel before reaching O, this wave will reach the observer after the interval $T_\omega=t_\omega+(P_\omega O/c)$.

Again from the geometry of the figure it will be clear that $PP_\omega+P_\omega O>PO$. Further, since $PP_\omega>c(t_\omega-t)$, it follows that $t_\omega-t+(P_\omega O/c)>PO/c$, or $t_\omega+(P_\omega O/c)>t+(PO/c)$, so that $T_\omega>T$. Hence it follows that $T_\omega>T>T_0$.

Thus an observer situated as indicated in Fig. 4 in relation to the trajectory of a projectile traveling throughout more slowly than sound will hear the various sounds resulting from the firing of the gun in the order of their production. The sound first heard will be the report of the gun. This will be followed immediately by a shrill continuous whistle, the pitch of which declines gradually at the outset, then drops markedly and finally declines gradually, terminating in the sound of the impact.

This sequence of events applied to all types of weapon up to the time of the Crimean War. Seasoned soldiers of that campaign were able to avoid trouble by taking appropriate evasive action in response to the warning con-

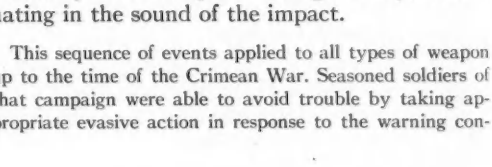

FIG. 4. Trajectory of projectile; $v<c$

veyed by the sound of the firing of a gun. Nowadays it applies only to arms of the howitzer and mortar class. A more homely illustration is provided by the sky rocket. However, as this is generally projected more or less vertically upwards and away from the observer, and explodes near the summit of its trajectory, the Doppler effect it exhibits is rather less striking.

Velocity of Projectile Greater Than That of Sound

Next suppose the projectile to be traveling faster than sound. Again, for simplicity, we start by assuming the trajectory to be a horizontal straight line along which the projectile has a constant velocity $v[>c]$. Then, as shown in Fig. 5, after the time t during which the projectile travels from a point p to P, the sound energy generated during its passage through p will be localized over a spherical wave front of radius pO, where $pO[=ct]$ is less than $pP[=vt]$. Let P_1, P_2, $\cdots$ represent the points along Pp occupied by the projectile 1 sec, 2 sec, $\cdots$ prior to its arrival at P. Then spheres described with centers P_1, P_2, $\cdots$ and with radii $P_1Q_1=c$, $P_2Q_2=2c$, $\cdots$ will represent the wave fronts over which the sound energy emitted as the projectile passes in turn through $\cdots$, P_2, P_1 is located at the instant the projectile itself reaches P. The members of this family of wave fronts do intersect one another, and it is possible, taking this point P as vertex, to draw a surface that touches each member of this family of spheres. This surface, being the envelope of the respective wave fronts at the instant the projectile passes through P, is a cone of semivertical angle α, where $\sin\alpha=P_1Q_1/P_1P=c/v$; this cone is a locus of concentration of sound energy. It affords a three-dimensional analog of the pair of diverging bow waves set up by a ship moving through water faster than the individual waves created by its passage are capable of traveling over the water.

The modern technic of high speed spark photography reveals this thin conical sheet of highly compressed air diverging from the head of a shell traveling with a velocity exceeding that of sound in air. The apex angle of this cone will remain constant as long as the velocity of the projectile is constant and will increase as this velocity decreases. The velocity component along the normal to the conical surface, however, remains equal to c throughout. This conical surface diverging from the head of the projectile and transported bodily along with the velocity v of the projectile itself will define the boundary of the sound field created by the projectile waves. Accordingly, an observer at O can receive no audible intimation of the presence of the projectile until the conical surface accompanying it passes over him; then, as is clear from Fig. 5, the projectile itself will have already passed beyond him. With the majority of present-day projectiles, ducking, far from being the equivalent of dodging, is merely a precautionary measure against more to follow.

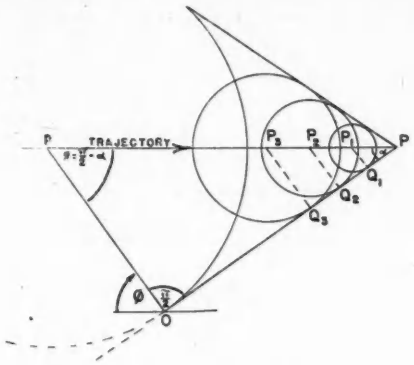

FIG. 5. Projectile-wave fronts. Horizontal trajectory; $v>c$.

The first sound heard under these circumstances is that due to the reception of the shock wave. The latter, for a fixed position of the observer relative to the trajectory, will originate from a fixed point on the trajectory, namely, the point (p in Fig. 5) in which the normal to the conical surface through O intersects the trajectory. The position of p will, accordingly, be defined by the condition, angle $OpP=\phi=\frac{1}{2}\pi-\alpha$, or $\cos\phi=\sin\alpha=c/v$.

That the shock wave is, in fact, the first to be received at O can be shown as follows. Let P_ϕ in Fig. 6 represent the point of origin of the shock wave, and let its distance to the left of $P_{\pi/2}$, the point on the horizontal trajectory of the projectile vertically above O, be x_ϕ. Then, since the relation defining P_ϕ is $\cos\phi=c/v$ and since, from the geometry of the figure,

$$\cos\phi=x_\phi/[(x_\phi^2+h^2)]^{\frac{1}{2}},$$

where $OP_{\pi/2}=h$, we have $v^2x_\phi^2=c^2(x_\phi^2+h^2)$, or

$$x_\phi=h\Big/\left[\left(\frac{v^2}{c^2}-1\right)\right]^{\frac{1}{2}}.$$

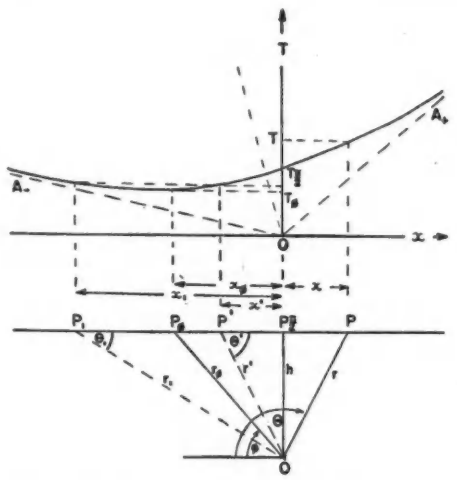

FIG. 6. T *vs.* x curve for horizontal trajectory; $v>c$.

Also

$$OP_\phi = r_\phi = [(x_\phi^2+h^2)]^{\frac{1}{2}} = (v/c)x_\phi = \frac{(v/c)h}{\left[\left(\frac{v^2}{c^2}-1\right)\right]^{\frac{1}{2}}}.$$

If the instant of passage of the projectile through P_ϕ is taken as the origin of time, the shock wave will arrive at O after a time $T_\phi = r_\phi/c$. The projectile wave from $P_{\pi/2}$ will arrive at O after a time $T_{\pi/2} = (x_\phi/v)+(h/c)$. The projectile wave from P, distant x to the right of $P_{\pi/2}$, will arrive at O after a time

$$T = \frac{x_\phi+x}{v}+\frac{r}{c}.$$

If these times are reckoned from the instant of passage of the projectile through $P_{\pi/2}$, the corresponding expressions become, respectively,

$$T_\phi = \frac{r_\phi}{c}-\frac{x_\phi}{v} = \left(\frac{v}{c^2}-\frac{1}{v}\right)x_\phi = \frac{h}{v}\left[\left(\frac{v^2}{c^2}-1\right)\right]^{\frac{1}{2}},$$

$$T_{\pi/2} = h/c,$$

$$T = \frac{x}{v}+\frac{r}{c} = \frac{x}{v}+\frac{[(x^2+h^2)]^{\frac{1}{2}}}{c}.$$

Now, since

$$\frac{1}{v}\left[\left(\frac{v^2}{c^2}-1\right)\right]^{\frac{1}{2}} < \frac{1}{c},$$

$T_\phi < T_{\pi/2}$, that is, the shock wave reaches O ahead of the projectile wave originating from the point on the trajectory vertically over the observer. This confirms the statement made previously that the projectile will have already passed beyond the observer before he receives any audible evidence of its presence.

The curve $T=f(x)$ is the hyperbola indicated in the graph forming the upper half of Fig. 6. The asymptotes to this hyperbola will be given by

$$T_{x\to\infty} = \frac{x}{v}+\left|\frac{x}{c}\right|,$$

where the absolute value of x needs to be taken in the second term since the corresponding term in the expression for T assumes the same value for pairs of values of x equal in magnitude but opposite in sign. These are indicated by broken lines on the graph, the asymptote OA_+ having the equation

$$T_+ = x\left(\frac{1}{c}+\frac{1}{v}\right),$$

and the asymptote OA_- having the equation

$$T_- = x\left(\frac{1}{c}-\frac{1}{v}\right).$$

Differentiation of T with respect to x gives

$$\frac{dT}{dx} = \frac{1}{v}+\frac{x}{c[(x^2+h^2)]^{\frac{1}{2}}},$$

which vanishes when

$$x = \frac{h}{\left[\left(\frac{v^2}{c^2}-1\right)\right]^{\frac{1}{2}}} = -x_\phi.$$

This particular value of x can be shown to determine a minimum of T, the value of which is

$$-\frac{x_\phi}{v}+\frac{[(x_\phi^2+h^2)]^{\frac{1}{2}}}{c},$$

that is, $(r_\phi/c)-(x_\phi/v)$, or T_ϕ, which establishes the point in question.

Since the point P_ϕ on the trajectory corresponds to the minimum value of T, points such as P_1 and P' can be located on either side of P_ϕ for which T has the same value. The projectile wave contributions from pairs of points so

situated in relation to P_ϕ will arrive simultaneously at the observer, thus increasing the intensity of the sound heard by him. Inspection of Fig. 6 shows that the distance of P' to the right of P_ϕ will invariably be less than the distance of P_1 to the left of P_ϕ.

The condition that T_1 shall be equal to T' is

$$\frac{r_1}{c}-\frac{x_1}{v}=\frac{r'}{c}-\frac{x'}{v},$$

which, on multiplying through by v/h, becomes

$$\frac{v}{c}\cdot\frac{r_1}{h}-\frac{x_1}{h}=\frac{v}{c}\cdot\frac{r'}{h}-\frac{x'}{h}.$$

Remembering that $\cos\phi=c/v$, we can express this in terms of the angles θ_1, ϕ and θ' in the form

$$\sec\phi\ \operatorname{cosec}\theta_1-\cot\theta_1=\sec\phi\ \operatorname{cosec}\theta'-\cot\theta',$$

an equation from which, when ϕ is known, θ_1 can be evaluated when θ' is given, or *vice versa*.

By way of example, let $v=2c$, that is, let $\phi=60°$, so that $\sec\phi=2$. The equation will then be of the form, $2\operatorname{cosec}\theta-\cot\theta=k$, or $2[(1+\cot^2\theta)]^{\frac{1}{2}}=k+\cot\theta$, the solution of which is $\cot\theta=\frac{1}{3}\{k\pm2[(k^2-3)]^{\frac{1}{2}}\}$. For real values of $\cot\theta$, then, we must have $k^2\geqslant3$.

When $k^2=3$, $\cot\theta=\sqrt{3}/3$, and $\theta=60°=\phi$. If k is large and if $\cot\theta_1=k$, then $\cot\theta'=-k/3$. A check on this is afforded from the ratio of the slopes of the asymptotes to the hyperbola of Fig. 6; this ratio is

$$\frac{(T_+/x)}{(T_-/-x)}=-\frac{(v/c)+1}{(v/c)-1}=-3=\frac{\tan\theta'}{\tan\theta_1}.$$

Thus, for example, if $\theta'=61°38'26''$, then $k=1.7329825$ and $\cot\theta_1=0.615542_5$, so that $\theta_1=58°23'9''$. And if $\theta_1=58°19'54''$, then $k=1.7330468$ and $\cot\theta'=0.538519$, so that $\theta'=61°41'48''$.

Again, if $\theta'=75°31'$, then $k=1.8073$ and $\cot\theta_1=0.9465$, so that $\theta_1=46°35'$; and similarly, if $\theta_1=41°24'$, then $k=1.8899$ and $\cot\theta'=0.1259$, so that $\theta'=82°49'$. The reason for this seemingly arbitrary choice of values of θ' and θ_1 will appear shortly.

On rotating Fig. 6 clockwise through a right angle it becomes applicable to a projectile descending vertically with its constant terminal velocity and bursting upon impact at $P_{\pi/2}$, at a horizontal distance h from the observer O. On the assumption that the speed of the projectile on impact exceeds that of sound in air, the interval elapsing between the reception of the shock wave and the burst wave will be

$$T_{\pi/2}-T_\phi=\frac{h}{v}\left\{\frac{v}{c}-\left[\left(\frac{v^2}{c^2}-1\right)\right]^{\frac{1}{2}}\right\}.$$

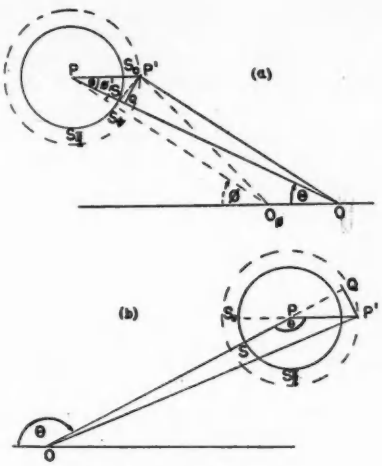

FIG. 7. Doppler effect; $v>c$.

Thus, for instance, if $v=2c$, then

$$T_{\pi/2}-T_\phi=(h/2c)(2-\sqrt{3})=0.134h/c;$$

that is, the burst wave will be heard 0.134 sec after the shock wave if h is 1120 ft and c is 1120 ft/sec.

A construction similar to that employed when $v<c$ can be applied when $v>c$ to evaluate ν', the observers' estimate of the projectile-wave frequency, in terms of its actual frequency ν. This is shown in Fig. 7, which is lettered to correspond with Fig. 2. Here, however, for a sufficiently small value of θ [Fig. 7(*a*)] the compression crest from P', or its equivalent Q, will arrive at O ahead of the compression crest from P, that is, in the reverse order of their production. Assuming it is still legitimate to regard the distance QS separating the two compression crests propagated towards O along PO as representing the wavelength λ' of the sound as received by the observer, we have $QS=PQ-PS$, that is,

$$\lambda'=\lambda\left(\frac{v}{c}\cos\theta-1\right)\quad\text{and}\quad\nu'=\frac{\nu}{(v/c)\cos\theta-1}.$$

When $\theta=0$,

$$\lambda_0'=P'S_0=PP'-PS_0=\lambda\left(\frac{v}{c}-1\right)$$

and

$$\nu_0'=\frac{\nu}{(v/c)-1},$$

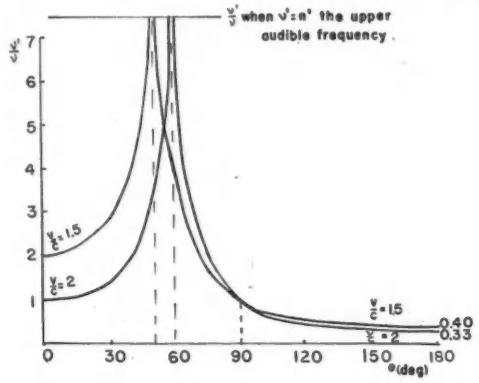

FIG. 8. Ratio of observed to actual frequency of projectile wave; $v>c$.

which agrees with the result obtained by direct substitution in the formula. If v/c lies between 1 and 2, then $\nu_0'>\nu$; if $v/c=2$, then $\nu_0'=\nu$; and if v/c exceeds 2, then $\nu_0'<\nu$.

Increase of θ from zero causes Q to approach S, thus decreasing QS, or λ', and so increasing ν'. When O reaches O_ϕ, a point so chosen that the arc of center O_ϕ and radius $O_\phi P'$ touches the circumference of center P and radius λ at the point S_ϕ, λ' will be reduced to zero and ν' will become infinite. The corresponding value, ϕ, of θ will be given by $(v/c)\cos\phi-1=0$, that is, $\cos\phi=c/v$; and, as we have seen already, for this particular value of θ, P is the point on the trajectory, relative to the observer at O_ϕ, from which the shock wave originates.

Increase of θ beyond ϕ will bring S ahead of Q on the line joining P to O. The compression crest from P will then arrive at O ahead of the succeeding compression crest from P', that is, in the order of their production, so that the original argument becomes valid again and

$$\nu'=\frac{\nu}{1-(v/c)\cos\theta}.$$

Further increase in θ will then increase λ' and so decrease ν'. When $\theta=\frac{1}{2}\pi$, $\lambda_{\pi/2}'=\lambda$ and $\nu_{\pi/2}'=\nu$, as before.

A precisely similar construction, indicated in Fig. 7(*b*), can be applied over the range of θ between $\frac{1}{2}\pi$ and π when the arc of center O and radius OP' intersects OP produced in Q and when $SQ=PS+PQ$. Then $\lambda'=\lambda[1+(v/c)\cos(\pi-\theta)]$, so that again

$$\nu_\pi'=\frac{\nu}{1+(v/c)}.$$

The manner in which ν'/ν varies with θ is exhibited in the two graphs of Fig. 8, corresponding, respectively, to values 1.5 and 2.0 of v/c.

Since there is an upper limiting frequency of the order of 20,000 sec^{-1}, above which the human ear is incapable of translating a vibratory aerial motion into the sensation of sound, the fact that the theoretical value of the projectile-wave frequency from points in the immediate vicinity of the point of origin of the shock wave itself is infinite implies that it is also inaudible. Accordingly, there will be an arc of the trajectory over which the projectile waves are inaudible, and this may be defined alternatively in terms of the corresponding angular interval $\Delta\phi$ in the neighborhood of ϕ. Of this interval a part, $\Delta\phi_1$, say, will occur before ϕ is reached; the remainder, $\Delta\phi'$, say, will occur after ϕ is passed.

If the upper audible frequency is denoted by n_1 or n', $\Delta\phi_1$ and $\Delta\phi'$ will be determined from the relations

$$n_1=\frac{\nu}{(v/c)\cos(\phi-\Delta\phi_1)-1}$$

and

$$n'=\frac{\nu}{1-(v/c)\cos(\phi+\Delta\phi')},$$

so that

$$\Delta\phi_1=\phi-\cos^{-1}\frac{c}{v}\left(1+\frac{\nu}{n_1}\right)$$

and

$$\Delta\phi'=\cos^{-1}\frac{c}{v}\left(1-\frac{\nu}{n'}\right)-\phi.$$

Thus, for example, taking

$$\nu=1000\text{ sec}^{-1},\quad n_1=n'=20{,}000\text{ sec}^{-1},$$

and $v/c=2$, that is, $\phi=60°$, we have

$$\Delta\phi_1=60°-\cos^{-1}0.525=60°-58°19'54''=1°40'6'',$$
$$\Delta\phi'=\cos^{-1}0.475-60°=61°38'26''-60°=1°38'26'',$$

and therefore $\Delta\phi=\Delta\phi_1+\Delta\phi'=3°18'32''$.

On the other hand, if $\nu=10{,}000$ sec^{-1},

$$\Delta\phi_1=60°-\cos^{-1}0.75=60°-41°24'=18°36',$$
$$\Delta\phi'=\cos^{-1}0.25-60°=75°31'-60°=15°31',$$

and therefore $\Delta\phi=34°7'$.

We have already seen that when $v=2c$ and $\phi=60°$, the projectile wave from a point on the trajectory corresponding to $\theta=\theta'=61°38'26''$ will arrive at O simultaneously with the projectile wave from a point on the trajectory corresponding to $\theta=58°23'9''$. The latter contribution, however, being still above the audible frequency limit, will be inaudible. Hence the first sound heard by the observer after the shock wave is a single note starting at a frequency equal to the upper limiting audible frequency from a point on the trajectory defined by $\theta=61°38'26''$ and falling in pitch as θ increases.

We have also seen that the projectile wave from a point on the trajectory corresponding to $\theta=61°41'38''$ will arrive at O simultaneously with the projectile wave from a point on the trajectory corresponding to $\theta=\theta_1=58°19'54''$. Both these component projectile waves will be audible, the former having a frequency below, and the latter a frequency equal to, the upper limiting audible frequency. Accordingly, the single note first heard will be succeeded by a complex sound consisting of two notes differing from each other in frequency and originating from points on either side of P_ϕ.

The projectile wave originating from the point on the trajectory to the left of P_ϕ will appear to the observer to have a higher frequency than that originating from the corresponding point on the trajectory to the right of P_ϕ. Thus, for example, the projectile waves from the pair of points defined, respectively, by $\theta_1=46°35'$ and $\theta'=75°31'$ have been shown to arrive simultaneously at O. Now for the former,

$$\frac{\nu_1}{\nu}=\frac{1}{2\cos 46°35'-1}=2.6695,$$

$2\cos 46°35'-1$ and for the latter,

$$\frac{\nu'}{\nu}=\frac{1}{1-2\cos 75°31'}=2.0000,$$

so that assuming, as before, $\nu=1000$ sec^{-1}, we have $\nu_1=2670$ sec^{-1} and $\nu'=2000$ sec^{-1}.

On the other hand, if $\nu=10{,}000$ sec^{-1}, then $\nu_1=26{,}695$ sec^{-1} and so is inaudible, while $\nu'=20{,}000$ sec^{-1} and is therefore just audible. The arc of inaudibility extends then in this case between values of θ defined by $\theta_1=46°35'$ and $\theta'=75°31'$. The sound first heard comes from the right of P_ϕ. It is a single note, and its initial frequency is 20,000 sec^{-1}. Not until $\theta'=82°49'$, when the corresponding frequency has fallen to 13,333 sec^{-1}, is any audible contribution received from the left of P_ϕ. The first contribution from this portion of the trajectory comes from a point defined by $\theta_1=41°24'$, corresponding to which the frequency is 20,000 sec^{-1}.

The interval between the instants of reception of the shock wave and the first projectile-wave contribution from the right of P_ϕ will be given by[3] $t'=(h/v)(k'-\tan\phi)$, so that if, for example, $h=1120$ ft and $v=2c=2240$ ft sec^{-1}, we have, since $\theta'=75°31'$ and $k=1.8073$, $t'=0.038$ sec. The interval between the instants of reception of the shock wave and the first projectile-wave contribution from the left of P_ϕ will be given by $t_1=(h/v)(k_1-\tan\phi)$, so that, since $\theta_1=41°24'$ and $k_1=1.8899$, we have $t_1=0.079$ sec.

Accordingly, the shock wave will, under the conditions postulated, be followed by an interval of silence lasting 0.038 sec, then by a single note, the frequency of which falls from 20,000 to 13,333 sec^{-1} in the course of the next 0.041 sec, and thereafter by a double note. The frequencies of the individual components of the latter will continue to fall from their respective initial values 20,000 and 13,333 sec^{-1} as θ_1 decreases and θ' increases.

The shock-wave sound is likely to be so intense as temporarily to deafen the observer, while the interval of silence succeeding it is likely to be too short to enable the ear fully to recover its sensitivity. Hence the shock-wave sound may to some extent mask the projectile-wave sounds immediately following the interval of silence. Moreover, as these particular projectile-wave sounds have frequencies near the upper limit of audibility, they will need to be fairly intense in order to produce an appreciable impression upon the ear.

The projectile-wave pitch corresponding to $\theta=0$ may be either above or below the true projectile-wave pitch. Thus, as shown in Fig. 8, when $v/c=1.5$, the former is an octave above the latter, and when $v/c=2$, the former will be in unison with the latter. The projectile-wave pitch corresponding to $\theta=\pi$ will invariably be below the true projectile-wave pitch. Thus when $v/c=1.5$, the former is a major tenth below the

[3] For $t=T-T_\phi=\left(\frac{r}{c}-\frac{x}{v}\right)-\left(\frac{r_\phi}{c}-\frac{x_\phi}{v}\right)$

$$=\frac{h}{v}\left[\frac{v}{c}\left(\frac{r}{h}-\frac{r_\phi}{h}\right)-\left(\frac{x}{h}-\frac{x_\phi}{h}\right)\right]$$

$$=\frac{h}{v}[\sec\phi(\operatorname{cosec}\theta-\operatorname{cosec}\phi)-(\cot\theta-\cot\phi)]$$

$$=\frac{h}{v}(k-\tan\phi).$$

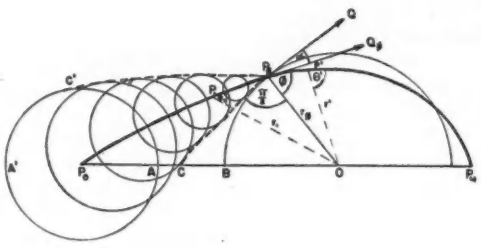

FIG. 9. Trajectory of projectile; $v>c$.

latter, and when $v/c=2$, the former will be a twelfth below the latter.

If the gun is located at a point P_0 on the trajectory to the left of $P_\phi(\theta_0<\phi)$, the corresponding point on the trajectory to the right of P_ϕ for which $T=T_0$ will be P_0'; and if the projectile strikes at a point P_ω to the right of $P_\phi(\theta_\omega>\phi)$, the corresponding point on the trajectory to the left of P_ϕ for which $T=T_\omega$ will be $P_{\omega 1}$.

If the report is heard by the observer before the impact, that is, if $T_0<T_\omega$, the double note resulting from the simultaneous reception of the projectile-wave sounds from points along the arcs $P_\phi P_0$ and $P_\phi P_0'$ will be followed, after the report, by a single note of falling pitch due to the reception of the projectile-wave sounds from points along the arc $P_0'P_\omega$ terminating in the impact. If the impact is heard by the observer before the report, that is, if $T_\omega<T_0$, the double note resulting from the simultaneous reception of the projectile-wave sounds from points along the arcs $P_\phi P_\omega$ and $P_\phi P_{\omega 1}$ will be followed, after the impact, by a single note of falling pitch due to the reception of the projectile-wave sounds from points along the arc $P_{\omega 1}P_0$ terminating in the report. This assumes, of course, that the projectile waves from the remoter arcs of the trajectory do, in fact, arrive at O with sufficient intensity to be audible. It also assumes that the arc P_0P_ϕ is acoustically effective, which, in view of the rather peculiar physical assumptions involved in the deduction of the corresponding Doppler effect, may be open to some doubt.

Imagine now the restrictions so far imposed to be removed. Then a similar construction can still be applied to the actual curvilinear trajectory. In Fig. 9, the instant depicted is that at which the projectile, having started out from P_0, has reached P_ϕ, the point on the trajectory from which the shock wave relative to the observer at O originates. The envelope of the family of spherical wave fronts originating from the various points along the arc P_0P_ϕ of the trajectory at the instant the projectile itself arrives at P_ϕ will be a more complicated type of conical surface intersected by the plane of the diagram in the pair of arcs $P_\phi C, P_\phi C'$.

The plane section of the accompanying sound field will (ignoring possible reflecting surfaces between P_0 and O) be bounded towards the rear by the circular arc $C'A'$ of center P_0 and radius ct_ϕ, where t_ϕ denotes the time taken by the projectile in traveling from P_0 to P_ϕ. If it is assumed that the projectile has a constant speed $v[>c]$, then $P_0P_\phi=vt_\phi$. The speed v will, in fact, be variable; but, if the relation $v=f(s)$ expressing this speed as a function of distance s measured along the arc of the trajectory is known, the time t_ϕ can be calculated by the evaluation of the definite integral

$$t_\phi=\int_0^{s_\phi} ds/v.$$

Let $P_\phi Q_\phi$ be the tangent at P_ϕ to the trajectory, and $P_\phi Q$ the tangent at P_ϕ to the lower branch of the envelope. Then angle $QP_\phi Q_\phi=\alpha$, and angle $OP_\phi Q_\phi=\frac{1}{2}\pi-\alpha=\phi$. Now with center O and radius $OP_\phi=r_\phi$, describe a circle cutting the line joining the gun emplacement P_0 to the observer O in B. Then the shock wave will reach the observer at a time $P_\phi O/c$ after the projectile has passed through P_ϕ. Moreover, any sound originating at the point B will take precisely the same time to reach the observer, since $BO=P_\phi O$. Now during the interval t_ϕ occupied by the projectile in traveling from P_0 to P_ϕ, the muzzle wave will have traveled a distance $P_0A[=ct_\phi]$ along P_0O. But since A lies to the left of B, the muzzle wave will arrive at O later than the shock wave by the time interval AB/c. Furthermore, inspection of Fig. 9 shows that this time lag will increase if the observer shifts his station along OP_ϕ to a point nearer P_ϕ.

The general expressions giving the times of reception of the various sounds at the observer O will be: (i) for the muzzle wave, $T_0=r_0/c$; (ii) for the projectile wave from a point P_1 to the left of P_ϕ,

$$T_1=\int_0^{s_1}\frac{ds}{v}+\frac{r_1}{c};$$

(iii) for the shock wave,

$$T_\phi=\int_0^{s_\phi}\frac{ds}{v}+\frac{r_\phi}{c};$$

(iv) for the projectile wave from a point P' to the right of P_ϕ,

$$T' = \int_0^{s'} \frac{ds}{v} + \frac{r'}{c};$$

(v) for the impact wave,

$$T_\omega = \int_0^{s_\omega} \frac{ds}{v} + \frac{r_\omega}{c}.$$

When times are reckoned from the instant of arrival of the shock wave at O, the interval between the reception of the shock wave and the projectile wave from P_1 will be

$$T_1 - T_\phi = -\int_{s_1}^{s_\phi} \frac{ds}{v} + \frac{r_1 - r_\phi}{c}.$$

But since $dr = -ds\cos\theta$,

$$\int_{r_1}^{r_\phi} dr = -\int_{s_1}^{s_\phi} ds\cos\theta,$$

that is,

$$\frac{r_\phi - r_1}{c} = -\int_{s_1}^{s_\phi} \frac{\cos\theta}{c} ds;$$

whence it follows that

$$T_1 - T_\phi = \frac{1}{c}\int_{s_1}^{s_\phi} \left(\cos\theta - \frac{c}{v}\right) ds.$$

Hence over that portion of the arc P_0P_ϕ for which $\cos\theta > c/v$, $T_1 - T_\phi$ will be positive and $T_1 > T_\phi$. Now normally over the whole of this arc, $\theta < \phi$ and $v > v_\phi$, where v_ϕ denotes the velocity of the projectile at P_ϕ. When the integrand is rewritten in the form $\cos\theta - \cos\phi v_\phi/v$, it is clear that as $\cos\theta > \cos\phi$ and, *a fortiori*, as $\cos\theta > \cos\phi v_\phi/v$, $\cos\theta - (c/v)$ will remain positive throughout the entire arc P_0P_ϕ.

The interval between the reception of the shock wave and the muzzle wave will be

$$T_0 - T_\phi = \frac{1}{c}\int_0^{s_\phi} \left(\cos\theta - \frac{c}{v}\right) ds = \frac{r_0 - r_\phi}{c} - t_\phi,$$

where

$$t_\phi = \int_0^{s_\phi} ds/v,$$

as we have already seen. The interval between the reception of the shock wave and the projectile wave from P' will be

$$T' - T_\phi = \int_{s_\phi}^{s'} \frac{ds}{v} + \frac{r' - r_\phi}{c}$$

and, since

$$\int_{r_\phi}^{r'} dr = -\int_{s_\phi}^{s'} ds\cos\theta,$$

that is,

$$\frac{r' - r_\phi}{c} = -\int_{s_\phi}^{s'} \frac{\cos\theta}{c} ds,$$

it follows that

$$T' - T_\phi = \frac{1}{c}\int_{s_\phi}^{s'} \left(\frac{c}{v} - \cos\theta\right) ds.$$

Hence, over that portion of the arc $P_\phi P_\omega$ for which $c/v > \cos\theta$, $T' - T_\phi$ will be positive and $T' > T_\phi$. Now normally over the whole of this arc, $\theta > \phi$, that is, $\cos\phi > \cos\theta$, while $v < v_\phi$, at least until the summit of the trajectory is passed. Accordingly, for at least a portion of the arc $P_\phi P_\omega$ immediately to the right of P_ϕ, $(v_\phi/v)\cos\phi - \cos\theta$ will be positive, and hence over this particular portion of the arc, $T' > T_\phi$.

The velocity of the projectile may pass through a minimum value on the descending branch of the trajectory, but usually in practice this minimum is not reached before the projectile hits its target. For values of $\theta > \frac{1}{2}\pi$, when $\cos\theta < 0$, the integrand will necessarily be positive. Thus the condition $T' > T_\phi$ will generally hold over the entire arc $P_\phi P_\omega$.

The interval between the reception of the shock wave and of the impact wave will be

$$T_\omega - T_\phi = \frac{1}{c}\int_{s_\phi}^{s_\omega} \left(\frac{c}{v} - \cos\theta\right) ds.$$

The muzzle wave may reach the observer before, simultaneously with, or after the impact wave. The muzzle wave will reach the observer before the impact wave if $T_0 < T_\omega$, simultaneously with it if $T_0 = T_\omega$, and after it if $T_0 > T_\omega$.

Corresponding points on the trajectory on either side of P_ϕ, the projectile waves from which will arrive simultaneously at O, will be determined by the condition $T_1 = T'$, that is,

$$\int_{s_1}^{s'} \left(\cos\theta - \frac{c}{v}\right) ds = 0.$$

If v is constant, this becomes

$$-\int_{r_1}^{r'} dr = (c/v)\int_{s_1}^{s'} ds,$$

or

$$\frac{r_1}{c}+\frac{s_1}{v}=\frac{r'}{c}+\frac{s'}{v},$$

as it obviously should.

Application to Sound Ranging

Of the three characteristic types of aerial wave generated by the projectile—(i) as it quits the muzzle of the gun, (ii) during its passage through the air, (iii) on impact at the target—the first alone is essential for sound ranging purposes. The other two are, however, by no means irrelevant, provided they are clearly distinguishable from the former and from each other on the record, since they afford valuable supplementary information. Thus the absence of the shock wave and the reception of the muzzle and impact waves in the order of their production indicates a low velocity projectile, as from a howitzer. The presence of the shock wave indicates that the initial velocity of the projectile exceeds that of sound. The interval between the instants of reception of the shock and muzzle waves may furnish some clue as to the caliber of the gun concerned.

An independent estimate of the caliber of the gun can also be read off from existing tables when the range of the gun and the time of flight of the projectile it discharges are known. The range can be found by plotting the points P_0 (located by sound ranging) and P_ω (located by direct observation) on the map and measuring their distance apart. The time of flight of the projectile can be found as follows. The sound of the report will reach the observer $r_0/c=T_0$ sec after the gun is fired. The sound of the impact will reach the observer $t_\omega+(r_\omega/c)=T_\omega$ sec after the gun is fired, where t_ω is the required time of flight of the projectile. Measurements carried out on the sound record secured by the observer enable the interval $T_\omega-T_0$ between the times of arrival of the muzzle and impact waves at O to be determined. And, since

$$T_\omega-T_0=t_\omega+(r_\omega/c)-(r_0/c),$$

we have

$$t_\omega=(T_\omega-T_0)+(r_0/c)-(r_\omega/c).$$

The Birth and Early Infancy of X-Rays

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The New Era in Physics

X-RAYS were discovered by Wilhelm Conrad Roentgen[1] at Würzburg, Germany, on November 8, 1895. The discovery amazed and excited both physicists and the general public. The newspapers vied with each other in reporting wild rumors, extravagant claims and fanciful speculations. The furor was spread over the first three months of 1896, reaching its zenith in February of that year.

This article has been written to commemorate the fiftieth anniversary of the discovery of x-rays. November 8, 1895, may be taken as the date of the birth of a New Era in physics. Before this notable date, it may be said, physicists were living in the Classical Era of physics. There were prenatal symptoms in the ten years preceding the birth of the New Era. Two such symptoms were the results of the first Michelson-Morley experiment, which were published in the *Philosophical Magazine* in 1887, and the description of what is now known as the photoelectric effect, by Heinrich Hertz in the *Annalen der Physik und Chemie* of 1887 and by Wilhelm Hallwachs in the *Annalen* of 1888. But these symptoms were not sufficient to cause the physicists to realize that anything startlingly new was approaching. Had not Hertz, in a previous issue of the *Annalen* of 1887, described the existence of electric vibrations in the ether and had not these been predicted by James Clerk Maxwell in his famous papers of the years 1861–65? There was nothing left to do except to measure every physical magnitude to the "next decimal place"!

The very disturbing cathode rays were another

[1] I shall use the spelling "Roentgen" throughout this article. A lesser physicist of Würzburg might be known as "Röntgen;" but the discoverer of x-rays belongs to the world; he is numbered amongst the illustrious dead in English-speaking countries. It is therefore appropriate that in these countries the anglicized form of the name of the discoverer should be used. We honor Roentgen in terms such as *roentgen*, a unit of x-ray dosage, and *roentgenologist*, an expert in x-rays. He is Roentgen.

symptom of the approaching birth. These rays were most puzzling. Sir William Crookes said they constituted a "fourth state of matter," but even he thought they were charged atoms or molecules. Many of the German physicists thought the cathode rays were some kind of disturbance in the ether. All physicists were clinging for dear life to the classical ideas of the existence of the luminiferous ether and the nonexistence of anything smaller in mass than the hydrogen atom or ion. A great jolt was needed to shake the minds of physicists loose from these ideas.

This jolt was supplied when the New Era of physics was born. The discovery of x-rays was quickly followed by that of radioactivity and by the isolation and recognition of the electron. Many other developments of the New Era have been announced from time to time until, finally, Hahn, Strassman and Meitner announced the discovery of nuclear fission in 1938–39. There is no reason, of course, to suppose that the developments of physics are at an end.

Almost two generations have now gone by since the discovery of x-rays. The younger physicists of today know very little of those exciting months of early 1896 just after the announcement was made. The older ones among us have personally known physicists, most of whom have now passed away, who remembered those stirring days. I have set myself four main tasks in the preparation of this article: first, to try to recapture some of the excitement, amazement and thrill of those early days and to describe the intense interest of the general public; second, to tell about Roentgen himself and his contributions to physics; third, to describe the world of physics into which the New Era was born; and, fourth, to use the facts of the early development of x-rays as an example of how physics advances.

For ease in reading, references are given in the text of the following sections. In this kind of article, dates are more important than volume and page numbers.

The Discovery of X-Rays

Perhaps the best account of the discovery of x-rays is given by H. J. W. Dam, in *McClure's Magazine* for April 1896. This periodical was similar to our present-day *Atlantic Monthly*. The opening paragraph of Dam's article is well worth quoting:

> In all the history of scientific discovery there has never been, perhaps, so general, rapid and dramatic an effect wrought on the scientific centers of Europe as has followed, in the past four weeks, upon an announcement made to the Würzburg Physico-medical Society, at their December meeting, by Professor Wilhelm Conrad Roentgen, professor of physics at the Royal University of Würzburg. The first news which reached London was by telegraph from Vienna to the effect that a Professor Roentgen, until then the possessor of only a local fame in the town mentioned, had discovered a new kind of light, which penetrated and photographed through everything. This news was received with a mild interest, some amusement, and much incredulity; and a week passed. Then, by mail and telegraph, came daily clear indications of the stir which the discovery was making in all the great line of universities between Vienna and Berlin. Then Roentgen's own report arrived, so cool, so businesslike, and so truly scientific in character, that it left no doubt either of the truth or of the great importance of the preceding reports. Today, four weeks after the announcement, Roentgen's name is apparently in every scientific publication issued this week in Europe; and accounts of his experiments, of the experiments of others following his method, and of theories as to the strange new force which he has been the first to observe, fill pages of every scientific journal that comes to hand. And before the necessary time elapses for this article to attain publication in America, it is in all ways probable that the laboratories and lecture rooms of the United States will also be giving full evidence of this contagious arousal of interest over a discovery so strange that its importance cannot yet be measured, its utility be even prophesied, or its ultimate effect upon long-established scientific beliefs be even vaguely foretold.

When Dam visited Roentgen's laboratory in January 1896, he was taken to a very small room whose walls were made of tin. In one wall there was a window of aluminum 1 mm thick. Outside the room were a Rhumkorff coil and a Crookes tube. The door was closed, and Dam was in black darkness. Dam held a sheet of paper coated with barium platino-cyanide between his face and the aluminum window. When the x-rays were turned on, the paper glowed with a yellow-green luminescence. Roentgen then told Dam to hold a book 2 in. thick between the aluminum window and the fluorescing paper. Dam could see no change in the brightness. The fluorescent paper was removed and, although the x-rays were still on, Dam could see nothing. As Dam describes his experience, the x-rays were invisible and impalpable and gave no sensation whatever. They were only to be judged by their effect on the fluorescent screen.

Dam apparently was put through a routine arranged by Roentgen for all newspaper and

magazine correspondents. It is strange that Dam was not instructed by Roentgen to place his hand between the aluminum window and the fluorescent screen. It was the photograph of the bones in the living hand, appearing in the public press, that had so excited the general public. Yet, Dam was not invited to "see" the bones in his own hand. However, Dam asked to see a photograph of a living hand, and Roentgen showed him various x-ray photographs.

Dam then interviewed Roentgen in part as follows:

> "Now, Professor," said I, "will you tell me the history of the discovery?"
>
> "There is no history," he said. "I have been for a long time interested in the problem of the cathode rays from a vacuum tube as studied by Hertz and Lenard. I had followed theirs and other researches with great interest, and determined as soon as I had the time, to make some researches of my own. This time I found at the close of last October. I had been at work for some days when I discovered something new."
>
> "What was the date?"
>
> "The eighth of November."
>
> "And what was the discovery?"
>
> "I was working with a Crookes tube covered by a shield of black cardboard. A piece of barium platinocyanide paper lay on the bench there. I had been passing a current through the tube, and I noticed a peculiar black line across the paper."
>
> "What of that?"
>
> "The effect was one which could only be produced, in ordinary parlance, by the passage of light. No light could come from the tube, because the shield which covered it was impervious to any light known, even that of the electric arc."
>
> "And what did you think?"
>
> "I did not think; I investigated. I assumed that the effect must have come from the tube, since its character indicated that it could come from nowhere else. I tested it. In a few minutes there was no doubt about it. Rays were coming from the tube which had a luminescent effect upon the paper. I tried it successfully at greater and greater distances, even at two meters. It seemed at first a new kind of invisible light. It was clearly something new, something unrecorded."

Note Roentgen's reply, "*I did not think; I investigated.*" This is one of the classic statements of physics. There has been much discussion over the years as to whether the discovery of x-rays was accidental or not. It is sometimes suggested that Roentgen was looking for some new phenomenon connected with cathode rays. Previous to his first paper on x-rays, he had, as we shall see later, published some 50 papers, none of which was on the Crookes tube or cathode rays. In the interview just quoted, Roentgen states that he began his experiments with a Crookes tube at the close of October 1895. On Friday, November 8, 1895, he "noticed" the luminescence of a fluorescent screen lying on a table. Dam's interview is of course a translation of Roentgen's words, and "noticed" may not be the best translation. However, I am indebted to Professor Arthur L. Hughes for lending me a copy of the January 28, 1896 issue of the *Münsche Medicinische Wochenschrift*, in which is given a report of a meeting of the Physico-medical Society of Würzburg held on January 23, 1896. The report indicates that Roentgen gave his first address on x-rays to the Würzburg Society on this date. In this address Roentgen said that in his experiments with cathode rays he noticed (*bemerkte*) the fluorescence of a sheet of paper painted with barium platino-cyanide and lying on a table, even though the Hittorf tube (German equivalent for Crookes tube) was completely enclosed in black cardboard. "Noticed" is a good translation of "*bemerkte.*"

Nothing is said in Dam's article or in the Munich weekly just mentioned about Roentgen's being led to the discovery of x-rays by the blackening or fogging of photographic plates placed near an excited Crookes tube. Many physicists working with Crookes tubes had found that photographic plates, though kept in light-tight boxes, became fogged and spoilt. It is said that Sir William Crookes even complained to the Ilford Company of London for sending him fogged plates. However, no one seems to have connected the spoiling of boxes of photographic plates with the proximity of these boxes to excited Crookes tubes. Since Roentgen had been working with a Crookes tube for perhaps no longer than two weeks, whereas other physicists had been using them for years, it is unlikely that he had observed any unusual photographic effect before he noticed the luminescence of the fluorescent screen lying on the table. I strongly lean towards the opinion that Roentgen's discovery of x-rays was accidental.

Roentgen's First Paper on X-Rays

According to an article on Roentgen by W. Friedrich in the *Physikalische Zeitschrift* (September 1, 1923), Roentgen delayed the announcement of his discovery for about a month. He wanted to be very sure, for he knew that the discovery was iconoclastic; it must shatter many cherished notions. Roentgen showed wisdom in

his patience. After considerable search through various histories of physics and also through obituary articles written at the time of Roentgen's death, I am doubtful that he actually read a paper before the Physico-medical Society of Würzburg in December 1895, as Dam, in the previously quoted article in *McClure's*, implies that he did. I can only assert that he communicated the article to the Society about a month after the discovery. He did address the Berlin Physical Society on x-rays on January 4, 1896, and he spoke before the Würzburg Society on January 23, 1896. His earlier communication to that Society was published late in December, 1895, in the *Proceedings* [*Sitzungsberichte*] of the Physico-medical Society of Würzburg.

An account of how in one case the news was learnt is given by Sir Arthur Schuster in *The Progress of Physics* (1911). In the first few days of January 1896, Schuster was returning to Manchester after a short Christmas vacation. On his way home he called at the laboratory for his mail and found a flat envelope containing photographs, which, without accompanying explanation, were unintelligible. Among them was one showing the outline of a hand, with its bones clearly marked inside. There was no letter, but there was a thin pamphlet entitled *Über eine neue Art von Strahlen*, by W. C. Roentgen. Schuster read and reread the pamphlet, keeping his family waiting in the cold outside. He was convinced, and in a day or so was taking x-ray photographs himself. It seems that Roentgen received reprints of his Würzburg article from the printers in late December and then proceeded to mail these pamphlets with accompanying photographs to certain physicists and medical men in Europe, England and America. These favored few then communicated the news by letter and telegraph to their colleagues and friends.

The delay with which the first news of the discovery was disseminated has been commented upon. It was due to the obscurity of the Würzburg journal and to slowness in mailing out the pamphlets. The first printed comment in a scientific journal in English on Roentgen's discovery appeared in the January 16, 1896 issue of *Nature*. This comment was reserved and somewhat skeptical. However, those who had received pamphlets were quickly convinced, and there was such a demand for Roentgen's original article that a translation was made by Arthur Stanton and published in *Nature* for January 23, 1896. It included a photograph of the bones of a living hand. Copies of the translation quickly appeared in *Science*, *Scientific American Supplement*, *Journal of the Franklin Institute* and the *Review of Reviews*. This last journal was in 1896 an American magazine somewhat similar to our present-day *Reader's Digest*. Reprints of the translation also appeared in some popular English magazines. To my knowledge this is the only case in which the original scientific article on a discovery in physics has been reprinted in any popular magazine. Another translation, made by G. F. Barker, is given by Professor E. C. Watson in an article appearing in the AMERICAN JOURNAL OF PHYSICS for October 1945.

It is surprising that Roentgen did not publish his first article on x-rays in the *Annalen*, which had a wide circulation and in which he had already published numerous articles. Most physicists in the English-speaking world read the translation in *Nature*. It seems likely that many Germans had to read the English translation. Finally, however, a reprint of the original article appeared in German in the January 1, 1898 issue of the *Annalen*. This is a very interesting fact. This reprint in the *Annalen* is dated December 1895. The same issue of the *Annalen* also contains a reprint, dated March 9, 1896, of Roentgen's second paper, which was originally published in the *Proceedings* of the Würzburg Society, and his third paper, dated March 10, 1897. The three papers in the *Annalen* occupy a total of 37 pages.

Newspapers and Public Reaction

At my suggestion, others have searched the principal New York, St. Louis and Los Angeles newspapers of late 1895 and early 1896 for news on x-rays. The first news found appeared on page 10 of the *St. Louis Post-Dispatch* of January 7, 1896. The news was from Vienna via London. The item with its headlines is worth quoting:

THE LIGHT THAT NEVER WAS

A Photographic Discovery Which Seems Almost Uncanny

A cablegram to the New York *Sun* from London says: The noise of war's alarms should not detract attention from the marvelous triumph of science which is reported from Vienna. It is announced that Professor Roentgen of Würzburg University has discovered a light which, for the purpose of photography, will penetrate wood, flesh and most other organic substances. The professor has succeeded in photographing metal weights which were in a closed wooden case and also a man's hand which shows only bones, the flesh being

invisible. . . . In contrast with ordinary rays of light these rays penetrate organic matter and other opaque substances just as ordinary rays penetrate glass. . . . The professor is already using his discovery to photograph bullets in human bodies.

The news from Vienna also stated that the professor used a Crookes tube and an induction coil. It was stated that broken bones could be photographed. The idea that by means of these new rays—x-rays—the bones of the hand could be photographed amazed and excited the public as had no other discovery in physics before this time (1896). As an example of the public interest, I shall quote some subsequent headlines in the *St. Louis Post-Dispatch*. My own abridgments of the news, and comments thereon, are shown in brackets.

January 31, 1896 Page 10

PHOTOGRAPHING YOUR BONES

Some authorities assert that the new power is electricity. Others believe that Roentgen has discovered a new form of energy, which probably works with a longitudinal and not a transverse vibration. . . .

February 2, 1896 Page 3

ROENTGEN LIGHT WONDERS

Discovery of Great Importance to Army and Navy

[Used to examine gun castings.]

February 6, 1896 Page 3

ROENTGEN'S PHOTOGRAPHS

[They can be produced with the aid of any static electrical instrument such as a Wimshurst machine. The light can be produced by using a tube without electrodes.]

February 7, 1896 Page 2

PROFESSOR ROENTGEN'S RAYS

Precious Stones Tested and Consumption Microbe Threatened

February 9, 1896 Page 2

ROENTGEN RAYS MADE LIGHT OF

Some Scientists Say They Have Known It All Along

Not a great discovery

February 11, 1896 Page 7

TO SEE THROUGH THINGS

Humans Can Look into a Closed Box

Invention by Professor Salvioni of Perugia enables eye to see x-rays.

February 12, 1896 Page 1, top left-hand corner

MORE ABOUT ROENTGEN RAYS

Edison Perfects a Tube for His Birthday Present

February 13, 1896 Page 1, top left-hand corner

HUMAN BRAIN PHOTOGRAPHED

Professor Roentgen Outdone by Dr. Simon of New York

Wholly new discovery

The Gotham physician illuminates his own gray matter and sees it pulsate. Picture taken without cathode ray. [This was pure extravaganza.]

February 14, 1896 Page 1, top right-hand corner

AND STILL THE WONDER GROWS

Roentgen's Discovery Is a Fundamental One

[Some scientists suggest that Roentgen's discovery may establish new theories of the world's creation. Salvioni's cryptoscope enables the human eye to see through opaque objects.]

February 16, 1896 Page 2

ON HUMAN AND CANINE SKULLS

Queer Mental Pictures

Photograph of a bone planted in a dog's brain which he afterwards found [*sic*]. [About this time humorists and cartoonists realized the usefulness of x-rays in their respective professions.]

February 21, 1896 Page 1, top left-hand corner

CATHODE RAYS WERE OBTAINED

Washington University Professors Repeat Roentgen's Experiments

Devised their apparatus

Professor Charles R. Sanger and Mr. A. S. Cushman succeeded Thursday night in photographing plainly the contents of a purse. [By cathode rays the newspaper meant x-rays. The picture was reproduced. This is an example of many such pictures which were being taken in the laboratories of various universities all over the world.]

March 13, 1896 Page 10

X-RAY EXPERIMENT

Some Results of Investigations at Princeton

Baron Roentgen Now

[Roentgen was made a baron with the privilege of using the aristocratic *von*.]

March 16, 1896 Page 3

EDISON'S NEW DISCOVERY

Can See Through Solids with Eye

Professor Roentgen feels slighted

[The third headline did not refer to the first two.]

After the middle of March 1896, public interest in x-rays began to subside and by the end of April 1896, x-rays were accepted as part of natural phenomena. It is interesting to note that news on x-rays rated the most important news spot (the top right-hand corner on page 1) of the *St. Louis Post-Dispatch* on February 14, 1896, which is about the time when the excitement ran highest in the United States. The stir probably reached its zenith somewhat earlier in Europe and England.

As we have seen, Roentgen was made a baron. Although sometimes referred to as "von Roentgen" in the early days of x-rays, he himself never used the title. Another point of interest is that in one news item Roentgen was described as feeling slighted. Roentgen was a quiet and modest man. Physicists, medical men and electrical engineers all over the world not only jumped on the band wagon, but some of them even played the loudest instruments in the band. Roentgen's quiet voice was drowned out. One newspaper writer said that Roentgen needed a press agent.

The headlines in the *St. Louis Post-Dispatch* are fair samples of those appearing in the Los Angeles and New York newspapers; the latter will not be given in detail, but a few items in these other newspapers will be mentioned. On February 16, 1896, the *Los Angeles Times* stated that developments in the new photography were multiplying too rapidly for record. In a matter of days, let alone weeks, experimental findings were superseded by others. Claims that x-rays could be focused were made, only to be rejected. It was reported that light from burning magnesium could penetrate wood and that pictures similar to x-ray pictures could be obtained without tubes. Professor Fernando Sanford of Stanford University stated in the *Los Angeles Times* of February 23, 1896, that

> . . . about two years ago German experimenters threw a bombshell into the English camp by showing that the cathode rays would pass through thin films or sheets of certain metals, and that by inserting an aluminum window in the glass they could even be made to pass out into ordinary air . . . , and Lenard . . . pointed out that the rays would produce photographic impressions. If he had followed this line up, Roentgen's discovery would have been antedated.

We thus see that along with much nonsense serious scientific discussion was appearing in the public press. Among the nonsense we find the following in another newspaper:

> It is suggested that, if all that has reached us by cable is true, there will no longer be any privacy in a man's home, as anyone armed with a vacuum tube outfit can obtain a full view of any interior through a brick wall.

Another news item suggested that an x-ray could bring back life and that cathode rays (there was confusion of cathode rays with x-rays amongst both the general public and the physicists) could be used for resuscitating electrocuted persons. A prominent electrical engineer claimed that x-rays or cathode rays were sound waves and that he had heard them. Another prominent electrical engineer set out to photograph the human brain, but his attempt was not a success. Bullets were being located by x-rays, and even toothache was dispelled by them. It was difficult to distinguish fact from fancy. One scientist claimed that he had been able to reflect x-rays from a sunbeam. X-ray pictures of snakes and frogs greatly interested the public. The St. Louis *Globe-Democrat* on March 29, 1896, stated that x-rays appeared to be harmful to the eyes. This was one of the first indications that x-rays might have such an effect on human tissue. The *New York Times* of March 15, 1896, in discussing cathode and x-rays, was moved to make the following remark:

> Whenever a discovery of an extraordinary character is made, it is seized upon by a multitude of writers, who, knowing nothing of the scientific principles involved, but being actuated by sensational tendencies, make claims for it that not only excel the actual accomplishments, but in many cases transcend the limits of possibilities. This has been to a very great extent the fate of Roentgen's x-rays.

We have seen how the possibility of photographing the human skeleton through the flesh amazed the public. The following is quoted from the *Scientific American* of February 22, 1896:

> The new photography has moved the English heart to poetry. The following verses are not by the new Poet Laureate, but they shed new light upon the future uses to which the shadow photographs may be put. Our thanks are due London *Punch*, to whom we are indebted:
>
> O Roentgen, then the news is true
> And not a trick of idle rumor
> That bids us each beware of you
> And of your grim and graveyard humor

We do not want, like Dr. Swift,
To take our flesh off and to pose in
Our bones, or show each little rift
And joint for you to poke your nose in.

We only crave to contemplate
Each other's usual full dress photo;
Your worse than 'altogether' state
Of portraiture we bar in toto!

The fondest swain would scarcely prize
A picture of his lady's framework;
To gaze on this with yearning eyes
Would probably be rated tame work.

No, keep them for your epitaph,
These tombstone souvenirs unpleasant;
Or go away and photograph
Mahatmas, spooks, and Mrs. Besant.

The Mrs. Besant of the poem was a prominent English theosophist (some people called her a spiritualist) of 1896. The English physicist, J. J. (afterwards Sir Joseph) Thomson, referred to *Punch* and probably to this poem at a meeting of the Royal Society.

Finally, Sir Arthur Schuster's description in *The Progress of Physics* of his experiences with the public in early 1896 is worth recording:

> My laboratory was inundated by medical men bringing patients, who were suspected of having needles in various parts of their bodies, and during one week I had to give the best part of three mornings to locating a needle in the foot of a ballet dancer, whose ailment had been diagnosed as bone disease. The discharge tubes had all to be prepared in the laboratory itself, and, where a few seconds exposure is required now [1911], half an hour had to be sacrificed owing to our ignorance of the best conditions for producing the rays.

Schuster states that such interruptions seriously interfered with his experiments on the magnetic deflection of cathode rays. Schuster's experiences were duplicated in many physics laboratories all over the world.

Wilhelm Conrad Roentgen

Roentgen was born on March 27, 1845, at Lennep in the Rhineland, Germany. He was an only child and spent most of his childhood in Holland, the native country of his mother. At the proper age he entered the Gymnasium at Utrecht, where towards the end of his course of studies he ran into trouble. Being caught in a youthful prank, he rather too emphatically refused to implicate others concerned. As a result he was expelled from the Gymnasium. This misfortune prevented his entrance to the Hochschule at Utrecht. However, he learned that it was possible to enter the Polytechnic at Zurich, Switzerland, without a certificate (*Maturum*). In 1863 he was accepted as a student in machine construction at Zurich. But he was no eager beaver, working only at his studies and in his laboratory. The beautiful lake with its surrounding mountains often drew him from the laboratory on lengthy excursions.

During his three years of study he attended lectures on mathematics and on the mathematical theory of heat, the lectures on the latter subject being given by Clausius. In 1866 he obtained the diploma of mechanical engineer. However, his interest was in experimental physics, and he continued his studies at Zurich under Kundt, the experimental physicist. He obtained the doctorate in 1869. He went with Kundt to Würzburg, Bavaria, in 1870, where the traditions seemed to irk him. In 1872 Kundt was called to Strasbourg and again Roentgen accompanied him. The University at Strasbourg had just been organized, and Roentgen was free from irksome tradition. In 1874 he was made a privat-dozent at Strasbourg, and all obstacles to his career were then removed. In 1875 he became professor of mathematics and physics at the Agricultural Academy of Hohenheim. In 1876 he returned to Strasbourg as professor of theoretical physics. In 1879 he was appointed professor of physics and director of the Physical Institute at Giessen, where he was very happy. In 1888 he was called to the chair of physics at Würzburg, as successor to Kohlrausch. He made his famous discovery of x-rays at Würzburg in 1895. In 1896 he, together with Lenard, was awarded the Rumford medal of the Royal Society of Great Britain. In 1898 he declined the chair of physics at Leipzig, as successor to G. Wiedemann.

In an obituary article in the September 1, 1923 issue of the *Physikalische Zeitschrift*, W. Friedrich of Freiburg describes Roentgen as being first of all an experimental physicist. He was the same type of physicist as Michelson, for his interest was in experiments requiring high precision. He was reluctant almost to the point of timidity to publish his results. Because he was so very careful, he never had to revise the experimental results reported in any of his papers. His attitude toward theoretical physics was like that of Faraday; he liked to think in terms of a mechanical model. At the time of his great discovery, he believed in the elastic-solid theory of the

ether. The idea of longitudinal waves in the ether was not abhorrent to his mind.

Friedrich states that Roentgen published a total of 59 papers, of which I have been able to identify all but two. Of these papers, three cover his famous work on x-rays; and from the end of 1897 till his death in 1923, he published only three more. His first paper on x-rays was by far the most important of the three on this subject. The other two are to be classified as being amongst many other papers on x-rays of equal or even greater merit.

Roentgen's extraordinary and brilliant work on x-rays was completed in a period of about 18 months in the fifty-first and fifty-second years of his life. After this period his active interest returned to those fields in which he had previously been interested. It is astonishing that, having unlocked the door to the New Era in physics, he avoided after 1897 any further exploration into the new realm. The reason is a great mystery. We recall the illustrious names of J. J. Thomson, Barkla, W. H. and W. L. Bragg, von Laue and others. Roentgen was in his fifties and sixties while these men were doing such brilliant work in x-rays. Did the discovery of x-rays come too late in Roentgen's life? Thomson was not quite 38 years old and the others were even younger at the time of the discovery. Was the discovery too iconoclastic for Roentgen's mind? There is quite a lot to be said for the view that physics is a young man's science. An open mind is needed. Cherished notions and too strongly held conceptions may retard the advance of physics. Perhaps the iconoclastic discoveries of physics and the development following these discoveries must be made by the younger physicists. But not all discoveries are iconoclastic.

In 1900 Roentgen left Würzburg to become professor of experimental physics and director of the Physical Institute at Munich. In 1901 he received the first Nobel prize in physics. Friedrich says that, although Roentgen was aware of the practical value of his discovery, he held himself remote from the fast-developing technology of x-rays. It was not in conformity with his modest scientific nature to exploit in a monetary way the material side of his discovery. Although we may admit his modesty and unworldliness, it is still a mystery why his scientific instinct did not compel him to do further work on x-rays after 1897. Friedrich says that as Roentgen became older and as his reputation became greater throughout the world, he turned all the more from this publicity back to the companionship (*Bedauern*) of the world of nature. While at Munich he spent much of his free time in the beautiful mountain village of Weilheim, where he owned a cottage. Although with his colleagues he was reserved almost to the point of unfriendliness, he invited his students to the cottage where he and they chatted and played ninepins. In this way his students came to know Roentgen as a man. Roentgen satisfied his love of nature by hunting game in the mountains surrounding his cottage.

Roentgen's wife died after a long illness in 1919. He retired from his chair at Munich in the spring of 1920. On July 22, 1920, his last article—on photoconductivity—was sent to the *Annalen*. This article runs to 195 pages in the January 1, 1921 issue. It was written when he was 75 years of age, and is one of his outstanding contributions to physics. After a short illness, Roentgen died on February 10, 1923.

Roentgen's Contributions Outside X-Rays

We have seen that Roentgen published some 50 papers before his discovery of x-rays. His first paper, entitled "A determination of the ratio of the specific heats of air," appeared in the *Annalen* in 1870. His thesis paper, entitled "Determination of the ratio of the specific heats at constant pressure and constant volume for several gases," appeared in 1873. As time went on he published papers in various fields of physics and applied physics. He described a telephone alarm in 1878 and a new aneroid barometer in 1879. It is interesting to note that in 1878 he wrote a paper on the discharge of electricity through crystals. He wrote on the Kerr effect, the Poisson ratio for rubber, the effect of an electric field on the double refraction in quartz. In 1883 he wrote on the thermo-, actino-, and piezoelectric properties of quartz. Then came papers on the effect of pressure on the viscosity of liquids, absorption of heat by water vapor, the thickness of oil films on water, the compressibility of carbon bisulfide and several other liquids, the constitution of liquid water (Roentgen believed that ordinary water contained dissolved ice), the effect of the heat of compression on the compressibility of liquids, the compressibilities of sylvine and rocksalt and of water solutions of these substances. In 1891 he wrote on the effect of pressure on the refractive indices of water, carbon bisulfide and several other liquids. Roentgen's last two papers before his discovery of x-rays appeared in the June 1, 1894 issue of the *Annalen*. One was on optical

experiments with a rectangular glass prism, and the other on the effect of pressure upon the dielectric constants of water and ethyl alcohol. Both these papers were dated February 1894. It was at the close of October 1895 that he began experimenting with a Crookes tube.

Summing up, one may say that, although Roentgen's experiments before his great discovery were in various fields, yet his main interests were in the physical properties of crystals and in the effect of pressure upon the physical properties of various substances. According to some physicists, von Laue amongst them, Roentgen's most important experiment, outside of his x-ray experiment, was performed in 1888. This experiment is not included amongst those mentioned in the preceding paragraph. In 1875 H. A. Rowland performed his famous experiment on the magnetic effect of electric convection (carrying of charges by moving, gross matter). The results were published in 1878. Other physicists were not uniformly successful in obtaining Rowland's results. In 1888, Roentgen undertook to repeat Rowland's experiment with apparatus of high precision. He not only confirmed Rowland's results, but was also able to obtain quantitative agreement between his experimental results and the results calculable from the Faraday-Maxwell theory. Roentgen published the account of this most important work in the *Proceedings of the Berlin Academy* and not in the *Annalen*, which, to say the least, had the wider circulation. The writer has been unable to find an abstract of this paper in the *Beiblätter*. It would seem that not only was Roentgen timid in regard to the publication of his most important results but that he was indifferent. In this respect he was similar to Cavendish (1731–1810). The importance of this paper is attested by the fact that the magnetic effect of convected charges is sometimes referred to as the Rowland-Roentgen effect. There is a relation here somewhat similar to that between J. J. Thomson and R. A. Millikan. Thomson obtained a qualitative result for the charge on the electron, while Millikan, by using methods of higher precision, obtained a highly accurate value for the charge. Roentgen's work on convected charges corresponds in a general way to Millikan's work on the electron.

As Friedrich says, Roentgen's work on x-rays was outside the range of most of his experiments. But so also was his work on convected charges. In both cases he seems to have departed from the field of his general interest for a year to 18 months and then to have returned to this field. After his discovery of x-rays, Roentgen published three papers on the following topics: determination of the thermal linear expansion coefficients of cuprite and diamond (1912), the piezoelectric effect in quartz (1913), and the effect of gamma-rays, x-rays and ordinary light on the electric conductivity of crystals (1921). Perhaps this last paper may be considered as being partly on x-rays, but Roentgen's interest was, as before, in a property of crystals. The subject of the last paper that Roentgen wrote would now (1945) be described as photoconductivity. This paper is the first one referred to in the chapter on photoconductivity in Hughes and Dubridge's *Photoelectric Phenomena*. Roentgen was thus one of the first, if not the first, to work on this phenomenon. It is worth recalling that in 1878 Roentgen wrote a paper on the discharge of electricity through crystals. There is no doubt that for 40 years or more he was interested in the physical properties of crystals.

Faraday on Physical Theory

Just after the discovery of x-rays there was great uncertainty as to the nature of the new rays. Some physicists held that they were of the same nature as Lenard rays, the rays that penetrate through an aluminum window in a Crookes tube to the air outside. The Lenard rays possess the same ability to affect fluorescent screens and photographic plates as do x-rays. Both Lenard rays and x-rays pass through matter that is opaque to ordinary light. We must remember that at the end of 1895 the electron had not yet been isolated. Electricity was something or some kind of a process in the ether which could be infinitely subdivided. This was the world of ideas into which x-rays were born. Before we proceed to a survey of this world, it is well to ponder the words of Faraday, printed in 1844 in the *Philosophical Magazine:*

> But it is always safe and philosophic to distinguish, as much as is in our power, fact from theory; the experience of past ages is sufficient to show us the wisdom of such a course; and considering the constant tendency of the mind to rest on an assumption, and, when it answers every present purpose, to forget that it is an assumption, we ought to remember that it, in such cases, becomes a prejudice, and inevitably interferes, more or less, with a clearsighted judgment. I cannot doubt but that he who, as a wise philosopher, has most power of penetrating the secrets of nature, and guessing by hypothesis at her mode of working, will also be most careful, for his own safe progress and that of

> others, to distinguish that knowledge which consists of assumption, by which I mean theory and hypothesis, from that which is the knowledge of facts and laws; never raising the former to the dignity nor authority of the latter, nor confusing the latter more than is inevitable with the former.

We shall see in the sequel how the advance of knowledge in physics is sometimes hindered or even prevented by established habits of thought. Physicists are prone to give too much veneration to theories that are successful. So far, no theory has been devised that is adequate for all purposes. There is always the possibility of another iconoclasm.

Cathode Rays

One is tempted at this place to speculate on what might have happened in the development of physics if in 1819 physicists had studied the passage of electricity through gases instead of through metal wires. As it was, however, the minds of physicists were pretty well steeped in Ohm's law by 1859. In that year, according to J. J. Thomson, in the second edition of his famous *Conduction of Electricity through Gases*, Plücker discovered cathode rays. Plücker published a series of seven papers in the *Annalen*, extending from 1858 to 1862. The first paper is entitled "On the action of a magnet upon the electric discharge in rarefied gases." Nowadays it is customary to speak of the Crookes dark space, next to the cathode. This space, however, is dark only by comparison with the brightness of the positive column. More exactly, the space is filled with a faint blue light. In the opening paragraph of Plücker's paper of 1859 we find: "The behavior of the light which is propagated from the negative electrode in all directions is one so peculiar that I shall give it further consideration." Plücker found that where this "faint blue light" extended to the glass walls of the tube, these walls glowed with a bright greenish-yellow phosphorescence. Further, he found that the greenish-yellow patches changed their position when a magnet was brought near to them, but that their deflection was not of the same nature as that of the rest of the discharge in the positive column when the pressure was high enough for a positive column to exist. Plücker's mind was fixed on the "faint blue light," and he wrote that this light could be deflected by a magnet. He spoke of it as "*magnetische Licht.*" In the paper of 1862, Plücker speaks of the electric radiation which in rarefied gases diverges from particular points on the negative electrode and travels to the glass walls of the tube. There seems to be a difference in Plücker's mind between the properties of a current already in being and one coming into being; thus, he disinguishes between the action of a magnet on a "*fertige Ströme*" and the action on a "*Ströme während ihres Enstehens.*" In the first case—perhaps that of a current in a metal wire—there is a sufficient supply of matter for the electricity to cling to; in the second case—that of a current in a rarefied gas—there is not a sufficient supply of matter. In a perfect vacuum there is no ponderable matter for the electricity to cling to, and so the ether should be a perfect insulator.

In the *Annalen* of 1869 Hittorf wrote that, although the process of conduction of electricity through solids and liquids, including electrolytes, was understood, yet the process in gases was one of the most obscure. Hittorf found that a solid body placed between a pointed cathode and the glass walls of the tube casts a well-defined shadow, the shape of the shadow depending only upon that of the body and not upon whether the latter be opaque or transparent, an insulator or a conductor. According to J. J. Thomson, Goldstein introduced the term *Kathodenstrahlen* in 1876. Goldstein regarded the cathode rays as waves in the ether. Nowadays, we are so used to the word *ray* in terms such as *alpha-ray* and *beta-ray* that we are apt to forget that in 1876 the word implied "radiation" or some process like light; to many physicists it did not imply "particles." Even in 1894, as we shall see, Lenard thought of cathode rays as a kind of radiation like light. In the *Proceedings of the Royal Society* of 1871, Varley put forward the view that the cathode rays were electrified molecules.

Crookes in the *Philosophical Magazine* of 1879 and Hittorf in the *Annalen* of the same year describe the mechanical and heating effects which accompany the cathode rays. Both speak of a mill with vanes that rotate under the action of the cathode rays. Crookes took the view that molecules coming from the neighborhood of the cathode give an impulse to the vane when they strike it. Crookes and Hittorf both knew that the cathode rays could be deflected by means of a magnet. Crookes concluded that the molecules were in a new form, or state, which he called the "fourth state of matter." It was thought that this fourth state appears only when the matter is highly attenuated; that, just as there is an abrupt change in the properties of a substance when it

changes from a solid to a liquid and from a liquid to a gas, so there might be an abrupt change in the properties of matter when it changes from a gas into the fourth state. Crookes took the view that in some way negative electricity is related to matter in the fourth state. In 1879, even though the facts of electrolysis were known, there was no concept of an "atom" of electricity, either positive or negative. The minds of physicists of that time were willing to accept the idea of positive and negative ions in electrolysis. The value of e/m for the hydrogen ion was conceived to be an average value for many ions. On the other hand, Hittorf's mind was on the faint blue light emanating from the neighborhood of the cathode. Although this light had peculiar properties, since it could be deflected by a magnet, nevertheless, according to Hittorf, it gave evidence of some process in the ether.

But Crookes' idea of charged molecules in the fourth state of matter was abhorrent to Hittorf, who, in the *Annalen* of 1883, vigorously stated that the transfer of a *fertige Ströme* (continuous current?) through a gas is not a sum of partial discharges which are separated from one another by small finite intervals of time and for which gas molecules are assumed alternately to be in a conducting and in a nonconducting state. Hittorf seems to have been bothered by the two concepts of electric charge and electric current. Some 30 years ago, when I was an instructor in an engineering school, current was more fundamental to the electrical engineers than charge. Electric charge was taught as $\int i dt$. In order to give a physical meaning to this integral, the current i must be continuous. This seems to be the way that Hittorf's mind worked. It must be remembered that in 1883 the magnetic effects of moving charged bodies had not yet been established. Rowland's results had not yet been fully accepted. The easiest way out, in Hittorf's mind, was to make the assumption that cathode rays were a process in the ether.

In the *Annalen* of 1892, Hertz describes experiments with a Crookes tube in which he found that cathode rays could pass through thin metal sheets in the tube. The cathode rays were both scattered in all directions and reflected. Hertz thought that perhaps the cathode rays passed through holes in the sheets of metal. By using three or four sheets, one behind another, he showed that the cathode rays passed through the sheets even though the sheets were slid along each other. This showed that holes were not responsible for the effect. The ability of cathode rays to pass through thin metal sheets prompted Lenard, Hertz's pupil, to see whether he could get cathode rays outside a Crookes tube and into the air. Lenard soon found that 15 layers of the usual aluminum foil would transmit cathode rays, and that a foil of sufficient thickness to withstand the pressure of the atmosphere would still transmit cathode rays. He placed the layers of foil over a small hole in the glass wall of the tube. The combination was air-tight. The cathode rays passed through the foil and into the air. The aluminum foil was grounded. This was the first time cathode rays were obtained outside a Crookes tube.

Lenard then set himself to make experiments on these rays outside the tube. He described his results in the *Annalen* of 1894 and 1895. We shall follow a frequent practice by calling these rays *Lenard rays*. The Lenard rays caused the air to glow. The glimmer was brightest near the aluminum window and extended in all directions, reaching to about 5 cm in air. Phosphorescent bodies lit up in air on the side presented to the aluminum window. The phosphorescence vanished at distances of 6 to 8 cm, but it was equally bright at the same distance from the window at all angles. A 0.5-mm quartz plate extinguished the Lenard rays, but the rays passed through thin gold, silver or aluminum foil. The opaqueness of quartz and the transparency of metal sheets were in contrast to the behavior of these substances toward light. If the beam of cathode rays in the tube was deflected from the aluminum window by a magnet, no Lenard rays appeared outside the tube. All solid substances more than 0.5 mm thick were opaque to Lenard rays, whether they were transparent to light or not.

Sharp shadows were cast on the phosphorescent screen when obstacles of thickness 0.5 mm were placed close to the screen. If the obstacle was moved away from the screen towards the window, the shadow became diffuse. According to Lenard the air is a turbid medium. The phenomenon is like that of light passing through milky water or through air laden with tobacco smoke.

The Lenard rays affect a photographic plate. There is no blackening behind a quartz plate. If an uncharged aluminum disk connected to an electroscope is placed in the beam of Lenard rays, no charge is shown by the electroscope. But, if the disk is charged, either positively or negatively, it loses its charge when placed in the path

of the Lenard rays. When the aluminum window was completely covered with a 0.5-mm quartz plate, no discharging action was found. But when the disk was removed to 8 cm from the window—that is, up to the boundary of the perceptible phosphorescence—the discharging action was not noticeably weaker. This action was still very noticeable at a distance of 30 cm from the window. When the hand was placed midway between the window and the disk, the discharging effect ceased. Commenting on these strange phenomena, Lenard wrote: "It cannot be decided whether we are observing an action of the cathode rays upon the surface of the irradiated aluminum window, or upon the air, or finally upon the irradiated disk! However, the last is very improbable at great distances from the window."

Both Hertz and Lenard found that cathode rays (inside the tube) could be spread by means of a magnet so as to produce a band of phosphorescence on the screen. This indicated a continuous spectrum and the existence of different kinds of cathode rays. Lenard concluded his 1894 article by saying that the processes in the ether which give rise to the cathode rays must be of such extraordinary fineness that dimensions of molecular orders of magnitude are involved. These dimensions are less than the smallest known wavelength of light, which, in 1894, was not less than 1000A. Lenard in 1894 persisted in the idea of the molecules of an attenuated space producing a turbidity in the ether.

In the *Annalen* for October 1, 1895, Lenard describes the use of an aluminum window for getting the cathode rays from a Crookes tube into an auxiliary tube which was so highly evacuated that no discharge could be made to pass between two electrodes in this tube with the potentials then available. Since the cathode rays —the faint blue light—were assumed to be processes in the ether made turbid by the presence of molecules, it was surprising that the rays continued to exist in a region where the attenuation was so much greater (pressure so much less) than in the ordinary Crookes tube.

We have already noted how Professor Sanford stated that German experimenters had thrown a bombshell into the English camp. In 1895 the German physicists, following Plücker and Hittorf, held to the view that cathode rays were some process in the ether. The rays might be "magnetic light." Yet they were light. These physicists knew that light could penetrate certain solid substances and that Hertz's electric waves could penetrate solid substances which are opaque to ordinary light. The Hertzian waves were a kind of light and so, they thought, were the cathode rays. Hence it did not seem so strange to Hertz and Lenard that the "magnetic light," or cathode rays, might penetrate metal sheets, which are opaque to ordinary light. On the other hand, the English physicists, following Crookes, held to the view that cathode rays were charged molecules something like the ions in electrolysis. In the English view, it was ridiculous to think of molecules passing through solids, whether these solids were transparent to ordinary light or not. The remarks of Faraday on physical theories are very pertinent here.

Lenard was the first physicist to study cathode rays outside the Crookes tube. As Professor Sanford implies, it is remarkable that Lenard did not stumble onto x-rays. Later, in 1896, Sir George Stokes also wondered why Lenard had not made the discovery. It was left to one outside the experimental field connected with Crookes tubes to discover x-rays. Thus it is that physics sometimes advances!

I have emphasized the German point of view because most English-speaking physicists are more familiar with the English, or Crookes, point of view. However, it should be remembered that the division of thought was not entirely on a nationalistic basis. There were Germans who supported the Crookes view and Englishmen who supported the Plücker-Hittorf view.

Finally, I shall mention a paper by Jaumann in the *Annalen* of 1895. This paper was on "Longitudinal light." There is no doubt that at the end of 1895 many physicists had their minds on the possibility of longitudinal vibrations in the ether. Jaumann's paper looks very imposing and is quite mathematical. Jaumann, along with many others, could not tear his attention from the faint blue light in the Crookes dark space. He compares the magnetic deflection of cathode rays with the magnetic rotation of the plane of polarization of light when it passes through carbon bisulfide. The direction of vibration of the longitudinal waves is caused to change in such a way that the waves proceed in a curved path. Jaumann arrives at this conclusion from a modification of Maxwell's equations. The modification disappears when the density of the attentuated gas in the Crookes dark space becomes zero and we have pure ether. Also it becomes unimportant in comparison with other effects when the pressure of the gas becomes atmospheric.

How X-Rays Advanced in 1896

The physicists of late 1895 and early 1896 passed through four fairly distinct stages in their views on x-rays. First, they were astonished and many of them incredulous; second, those who had an induction coil, Tesla coil or static machine and any kind of vacuum tube (vacuum in the sense used in 1896) rushed to their laboratories to try the experiments, only to find that Roentgen was right; third, having confirmed Roentgen's results and having read and digested the contents of his paper, the physicists of the day felt that Roentgen had so completely covered all phases of the phenomenon that there was not much left to do except make more accurate measurements; and, fourth, they gradually began to realize that Roentgen had by no means found out all there was to know about x-rays. With the coming of the fourth stage there began a period of advance in physics and of vigorous competition amongst physicists the like of which had never before been experienced. Developments came so fast that it was impossible to record all of them. On February 20, 1896, *Nature* complained:

> . . . so numerous are the communications being made to scientific societies that it is difficult to keep pace with them, and the limits of our space would be exceeded if we attempted to describe the whole of the contributions to the subject, even at this early stage.

Nature then proceeded to give quite a long review of x-rays up to that time. The subject index of the *Comptes Rendus* for the first six months of 1896 lists 130 papers under "Rayons X ou rayons Roentgen."

In order to fulfill my present purpose, I have conned the pages of the weekly journals, *Science*, *Nature*, *Comptes Rendus*, and *Naturwissenschaftliche Wochenschrift*, for they gave week-by-week descriptions of what was developing in x-rays. However, because of considerations of space, I have given considerable preference here to *Nature*, which in 1896 contained excellent reports of scientific meetings, both English and foreign. Because I have used *Nature* as my principal source, it may be that my discussion unintentionally gives undue weight to the work of English physicists. However, there were disagreements even amongst English physicists, and it is my hope that *Nature* gives a fair picture of the attitudes among the physicists of the world.

In a paper before the Paris Academy of Sciences on February 3, Benoist and Hurmuzescu reported that x-rays discharge an electroscope and that this offers a ready method for measuring the opacity of various substances to the rays. In a paper before the Royal Society on February 13, J. J. Thomson reported that a charged metal plate in air loses its charge when x-rays fall upon it and that this discharge occurs whether the plate is charged positively or negatively. The reader should remember that in 1896 ultraviolet rays were known to discharge negatively but not positively charged plates. Thomson believed that the x-rays in passing through the air split the molecules by a process similar to that in electrolysis. Thomson stated that the rate of discharge of an electrified plate gives a more delicate method of detecting x-rays than does the photographic method.

Roentgen also observed the discharging property of x-rays. This was reported in a communication to the *Sitzungsberichte* of the Würzburg Society. The date on the communication is March 9, 1896. A translation made by G. F. Barker is given by Professor Watson in his article in the AMERICAN JOURNAL OF PHYSICS for October 1945. The first paragraph of Barker's translation follows:

> *18.* It was known to me at the time of my first publication that x-rays can discharge electrified bodies; and I conjecture that in Lenard's experiments it was the x-rays, and not the cathode rays, which had passed unchanged through the aluminium window of his apparatus, which produced the action described by him upon electrified bodies at a distance. I have, however, delayed the publication of my experiments until I could contribute results which are free from criticism.

It is a pity that Roentgen did not include these further results in his first paper, or that, wishing to add to them, he did not write a letter to *Nature*. Instead, he continued to publish in an obscure journal. As we have mentioned, this article was finally reprinted in the *Annalen* of January 1, 1898. If Roentgen felt slighted because of the publicity received by others, he had only himself to blame. We honor him for wanting to be sure, but one is amazed at such timidity, modesty and perhaps indifference. All through this article I have adopted the principle, *De mortuis nil nisi verum*, rather than *De mortuis nil nisi bonum*. We should know the truth if we are to learn from past human experience.

There were definitely two schools of thought in regard to x-rays. One school held that x-rays were transverse vibrations in the ether and of the same nature as ordinary light. There was a difference of

opinion in this school as to whether the wavelengths of these vibrations were shorter or longer than those of any known ultraviolet or infra-red radiation. The other school believed that x-rays were cathode rays and that, following the ideas of Plücker, Hittorf, Hertz, Lenard and, finally, Jaumann, these cathode rays were longitudinal vibrations in the ether. Roentgen did not properly belong to either school, for he emphasized in his first paper the fact that x-rays could not be deflected by even the strongest magnetic field and so were different in nature from cathode rays. It is well to remember in this connection Faraday's previously quoted words on prejudices. Schuster, in a rather sarcastic letter in the January 23, 1896 issue of *Nature*, wrote:

> It seems that the idea that cathode rays are due to vibrations has become fashionable; yet the fact that the magnet deflects them just as it would an electrified molecule has always seemed to me to be conclusive against this view. No one has given, so far, any plausible reason why a ray of *invisible* light should be able to run around in a spiral while a ray of *visible* light goes straight; and, so far, Roentgen's rays behave as we should expect well-conducted vibrations to do.

Nevertheless, at the meeting of the National Academy of Science held in the spring of 1896, at Washington, A. A. Michelson maintained that x-rays were not essentially different from cathode rays.

J. J. Thomson, in his paper of February 13, stated that in order to obtain x-rays, both a cathode and a phosphorescent substance were needed. At that time Crookes tubes were pear-shaped, with the anode tucked away in a small side tube. The rod supporting the cathode was inserted at the position of the stem of the pear. The cathode was often flat and perpendicular to the axis of the pear. The glass of the bulbous part of the pear opposite to the cathode fluoresced when high voltage was applied to the tube. There was no metal target as there is in modern tubes. It was noted that a Crookes tube emitted x-rays only when phosphorescence (meaning fluorescence) occurred. So, from our vantage point of 1945, we can say that Thomson scored one hit and one miss in this paper. However, he also scored a second miss (or, at any rate, a near miss): he reported that a charged metal plate embedded in paraffin or sulfur lost its charge when x-rays fell upon it. A plate immersed in paraffin oil was also said to lose its charge. Thomson inferred that any dielectric, whether gas, liquid or solid, became conducting when traversed by x-rays. According to an article in *Nature* of June 4, Righi, after experimenting with dielectrics, dissented from Thomson's opinion that a nongaseous dielectric is rendered conducting when traversed by x-rays. This led to a long controversy which began to be resolved only when, in 1921, Roentgen wrote his last paper—the one on photoconductivity.

For a time there was quite a controversy over whether a Crookes or some other type of vacuum tube was needed to produce x-rays. Some claimed that rays from the spark of a static machine or from an electric arc would penetrate black paper and produce shadowgraphs of metal objects on a photographic plate. Sunlight was said to penetrate aluminum foil. Results were obtained in several of these experiments, but the results were spurious, that is, not due to x-rays. W. S. Franklin, of Iowa State College, took Silvanus P. Thompson, of London, to task in *Science* of March 6. Thompson had reported that with rays from the electric arc he had made shadowgraphs of metal objects on a photographic plate enclosed in a plate holder. Franklin cooled the plate holder and the aluminum and lead sheets placed on the plate-holder shutter by means of a blast of air. He obtained no shadowgraph and inferred that x-rays were not emitted by the arc.

Although it was quickly found that a vacuum tube was necessary, yet the tube did not need to be of the Crookes type. According to *Nature* of February 20, Wallace and Pockington, of the Leeds Central Higher Grade School, found that an ordinary incandescent lamp became a source of x-rays when the high frequency voltage from a Tesla coil was applied to a sheet of tinfoil on the outside of the lamp as one electrode and to the filament as the other. What we now call an electrodeless discharge had occurred, and the "vacuum" was just right for the production of x-rays.

In *Nature* of February 27, J. J. Thomson wrote that he could find no evidence of polarization when x-rays traversed parallel and crossed tourmaline plates. Perrin, Joly and Lodge had confirmed the lack of magnetic deflection of the x-rays, and so these rays must be different from cathode rays. Thomson found that when air which had been irradiated by x-rays was blown against a charged metal plate, the plate lost its charge even though the x-rays did not fall directly on the plate. The discharging action is therefore different from that occurring when ultraviolet light falls upon a negatively charged

plate. For the first time, Thomson spoke of the ionization of a gas. He conceived the process as being similar to the ionization of an electrolyte. He inferred that the rate of leak of the plate is proportional to the number of ions, provided that the degree of ionization is small.

Erskin Murray, by passing x-rays through air between plates of different metals, observed a kind of voltaic difference of potential similar to that obtained when the metals are immersed in an electrolyte. Thomson also reported that C. T. R. Wilson had found that x-rays produce dense clouds in a cloud chamber. Thomson found that when x-rays are passed through a small number of sheets of tinfoil, the addition of another sheet produces a considerable diminution in the rate of leak. However, when the number of sheets is fairly large, the addition of another sheet produces a smaller diminution of the rate of leak. Thomson therefore inferred that a beam of x-rays consisted of rays of different absorbability.

We have seen that, on February 11, the press credited Salvioni of Perugia, Italy, with being able to see x-rays. Salvioni's paper was read before the Perugia Academy on February 6. His paper created such a stir in the press that *Nature* of March 5 carried a translation. Salvioni disagreed with Roentgen on x-rays being longitudinal ether waves, and suggested that they were a highly rarefied state of matter projected from a Crookes tube. Using sheets of metals ranging in density from magnesium to platinum, he confirmed the relation found by Roentgen between transparency and density. Salvioni did not claim that the eye could see x-rays directly. However, he devised the cryptoscope, by means of which x-rays could be seen indirectly. This apparatus consisted of a cardboard tube 8 cm long, one end of which was closed by a fluorescent screen and the other by a lens. On looking through this cryptoscope, one could see metallic bodies enclosed in boxes of cardboard or wood. The cryptoscope could be used in a lighted room. Previously the fluorescence produced by x-rays could be seen only in a darkened room. It was the idea of being able to "see" x-rays in a lighted room that caused such a stir amongst the press and public. This stir was so great that Oliver Lodge and Alexander Gray were moved to write letters to *Nature*, on March 5, protesting against the tendency of sensational newspapers to praise Salvioni's "discovery" so highly as to diminish the credit due to Roentgen.

For the sake of brevity, dates given in the remainder of this section will refer to communications to *Nature* or to meetings reported in *Nature*. On March 5, A. W. Porter reported that a platinum disk hit by cathode rays became a source of x-rays. A week later, J. W. Gifford stated that the best tube for the production of x-rays was a spherical tube designed by Crookes in 1879 to show the incandescence of platinum under the impact of the projected molecules which were focused on it by a concave cathode. Roentgen stated in his first paper that he had obtained x-rays from an aluminum window. Thomson was thus mistaken in supposing that the cathode rays had to hit a phosphorescent substance to produce x-rays.

At about this time, concave cathodes and metal targets began to appear in x-ray tubes. At first it was thought that the target must be the anode, but this was found not to be necessary. As a result, the target was sometimes called the *anticathode*. However, on April 23 Thomson still doubted that the origin of x-rays is always at a place where the cathode rays strike a solid body. He believed that x-rays might originate in the gas of the Crookes tube. The pressure of the gas in a Crookes tube was 0.001 mm of mercury or less.

In the same paper of April 23, J. J. Thomson reported on the relation between the ionization current between two metal plates and the potential difference between the plates. Thomson found that, so long as the potential difference did not exceed 2 or 3 v, the ionization current was proportional to the potential difference. For larger potential differences the current increases faster than does the potential difference. Still larger potential difference brings about a condition in which the ionization current remains constant with increase of potential difference. Thomson noted that the graph of ionization current *versus* potential difference had the same general features as that of magnetization *versus* magnetizing force for soft iron. For chlorine gas he found the current to be the same for 10 v as for 278 v. Thus the idea of a saturation current was introduced.

At this time it began to be evident that the rate of discharge of a plate might lead to discordant results. Something like the modern ionization chamber began to be developed, and proper attention began to be paid to electrostatic shielding. Also, to obtain comparable results, it was found to be necessary to apply a potential

difference which was above that for saturation. Thomson and McClelland found that the saturation ionization current through a gas increased, in general, with the atomic weight of the gas. However, for solid dielectrics such as paraffin and sulfur, Ohm's law was valid up to 278 v and there was no evidence of saturation. The reader should remember that Righi and others dissented from this result for solid dielectrics.

According to Thomson, several experimenters had found copious "diffuse return" of x-rays from a solid obstacle placed outside the Crookes tube and in the path of the x-rays. At first it was believed that regular reflection had been obtained from speculum metal, but by proper arrangement of lead slits so as to shield the photographic plate or the electroscope from the direct rays it was found that diffuse return (that is, scattering) occurred in all directions. Discussing the matter of regular reflection, M. L. Pupin, of Columbia University, stated on April 6 that there could be no regular reflection of x-rays; otherwise powders would be opaque, and Roentgen, in Section 7 of his first paper, had found no difference in the transparency of a solid substance to x-rays whether it was powdered or not. Pupin said that x-rays are "diffusely scattered" when they pass through bodies, gases not excepted. He said that every substance irradiated by x-rays becomes a radiator of these rays.

FitzGerald and Trouton of Dublin were reported on April 30 to have looked for evidence of reflection of x-rays from crystals at grazing incidence, but without success. They found paraffin to be a good scatterer. On March 2, Imbert and Sans of Paris reported that the scattering of x-rays depends on the nature of the substance rather than on the condition of its surface. On August 13 it was reported that Dwelshauvers-Dery, of Belgium, had found that the scattered rays were more easily absorbed than were the incident x-rays. However, there was some doubt as to whether the scattered rays were x-rays. Roentgen, in Section 8 of his paper, had reported observing backward scattering from star-shaped pieces of platinum, lead, zinc and aluminum. He concluded that regular reflection did not exist but that bodies behave to x-rays as do turbid mediums, like milk in water, to light. On August 22, Sir George Stokes, in discussing the scattering of x-rays, stated that when the rays fall on a metal they throw the molecules into a state of vibration, which vibrations are then communicated to the ether. However, Stokes believed that these secondary rays may not be of exactly the same nature as the incident x-rays but that they "still have enough of the X quality about them, whatever that is, to enable them to get through objects which are opaque to ordinary light."

We have already described how J. J. Thomson, using parallel and crossed tourmalines, could find no evidence for the polarization of x-rays. A. M. Mayer, of Stevens Institute of Technology, used herapathatite (an iodo-sulfate of quinine), which he chose because of its low density, about 1.8 g/cm³. Crystals 0.05 mm thick were used. When the axes were at 90°, the crossed crystals were completely opaque to light, but were just as transparent to x-rays as when the axes were parallel. In a report on April 16, Frankland stated that the plane of polarization of optically active substances is not affected by x-rays.

On February 23, Gossard and Chevallier found that x-rays falling on a Crookes radiometer stopped the rotation of the vanes. Commenting on this result on March 23, J. R. Rydberg stated that the effect observed by Gossard and Chevallier had its origin in the well-known layer of positive electricity with which the external anticathodic surface of the Crookes tube is covered. Rydberg found that, on covering the radiometer with metal gauze, the effect observed by Gossard and Chevallier disappeared.

During 1896 knowledge concerning the absorption of x-rays did not advance much beyond the facts described by Roentgen in his first paper. Roentgen said that the absorption increases mainly with the density of the absorber. But he found that a plate of Iceland spar was more opaque than plates of glass, aluminum and quartz of similar thickness and of roughly equal densities. On March 12, J. S. McKay of Packer Institute, Brooklyn, reported that the relative opacity of substances to x-rays was in inverse proportion to their diathermancy. Thus rocksalt was most opaque, then alum, then glass, and then quartz. The terms *opacity*, *transparency* and *permeability*, as used in 1896, were too qualitative. The absorption coefficient k in the relation $I = I_0 e^{-kt}$ was not used in general. And, since J. J. Thomson had shown that the x-rays were heterogeneous, there was perhaps not much point in trying to determine k. On March 23, Bleunard and Labesse reported that the opacity of solutions of metallic salts increases with the atomic weight of both the metal and the nonmetal. On March 13, Ackroyd and Knowles reported that the opacity of metals, oxides, and sulfates increased with the molecular

weight, rather than the atomic weight. By the end of 1896 the situation as regards absorption of x-rays was confused, and this confusion lasted for several years. About all that could be said was that the absorption depended mainly on the density, with the atomic or molecular weight also having some effect.

Throughout 1896 the controversy over the natures of x-rays and of cathode rays raged. Roentgen in his first paper had hazarded the opinion that x-rays were longitudinal waves in the ether, even though they were not cathode rays. In 1896 the luminiferous ether held an important place in the thoughts of physicists, many of whom were expecting some evidence for the existence of such longitudinal waves, and Roentgen rather naturally felt that he had found something that pointed in this direction. But cathode rays were somehow bound up with x-rays. J. J. Thomson had said that they were related as parent to child. Lenard in his experiments had found that the cathode rays, after passing through an aluminum window into a highly evacuated auxiliary tube, were bent in varying degrees by a magnetic field, thus producing a continuous magnetic spectrum. Since some cathode rays were less deflected than others, there might be cathode rays that could not be deflected by a magnet; these nondeflectable cathode rays might very well be x-rays.

However, others, including Roentgen, refused to accept this explanation of the lack of deflection by a magnet. On March 5, Salvioni suggested that x-rays might be a highly rarefied form (state) of matter projected from a Crookes tube. On June 4, Goldhammer was reported to believe that x-rays were not longitudinal, but were ultraviolet rays of extremely short wavelength. The absence of reflection would then be due to the shortness of the wavelengths compared with the unevenness of ordinary polished surfaces. On the other hand, some physicists felt that x-rays must have a very long wavelength. Their ability to penetrate all substances suggested a similarity with Hertz's electric waves. It had recently been shown that Hertz's waves could pass through the walls of a house and the bodies of human beings up to a distance of three-quarters of a mile! Hence x-rays might have a wavelength between those of the infra-red rays and Hertz's waves. On August 22, Lodge stated that x-rays are transverse waves and that their wavelength is not greater than the size of atoms. On August 13, it was reported that an interesting attempt had been made by de Heen, a Belgian, to compute the wavelength of x-rays. De Heen quoted J. J. Thomson as finding a mean velocity of 3.3×10^7 cm/sec for the molecules in a Crookes tube. From the kinetic theory de Heen found that this corresponds to a temperature of 4.8×10^{7}°C! From Wien's displacement law, which had been published in the *Annalen* for 1893, this gave a wavelength, for maximum intensity, of about 1A.

Becquerel had first reported on the peculiar phosphorescence of uranium salts on February 24. These Becquerel rays, as they came to be known in 1896, added confusion to the controversy about the nature of x-rays. The Becquerel rays had powers of penetration similar to those of x-rays. They could be refracted and polarized but they could also be bent by a magnet. In the Rede lecture given on June 10 at Cambridge, J. J. Thomson referred to the Becquerel rays as light rays, possibly of wavelength between those for ultraviolet light and for x-rays. But the lack of refraction of x-rays caused Thomson to discuss anomalous and normal dispersion of light. The reader should remember that Lorentz' theory of refraction was unknown in 1896. In anomalous dispersion, the longer waves are more refracted than are the shorter, but in normal dispersion the reverse is true. There might conceivably be a region of wavelengths where no refraction occurs. Thomson said that this would be so if x-rays have a very high frequency compared with those of light waves. Thomson explained the lack of polarization of x-rays by tourmaline by supposing that the structure of tourmaline was not fine enough to affect the x-rays. This again indicated the smallness of the wavelength of x-rays. Also in 1896 it was suggested by other physicists that x-rays might be polarized cathode rays or that they might be vortices in the ether.

An interesting discussion took place between Sir George Stokes and Lenard at the meeting of the British Association, which was reported on October 8. Lenard insisted that cathode rays were disturbances in the ether. How else could they pass from the inside of a Crookes tube through an aluminum window to the air outside or into a highly evacuated auxiliary tube? Hertz's waves, which are admittedly ether waves, can pass through brick walls. Also, some substances, such as glass, are transparent to ordinary light. Why, therefore, should not aluminum be transparent to the "faint blue light"—the

cathode rays? This transparency of aluminum to cathode rays really had the Crookes school of physicists stumped.

Stokes got around this impasse by declaring that the molecules which hit the inside of the aluminum window were not the same molecules as those which left the window on the outside. The reader must remember that the electron had not yet been discovered. Stokes and other English physicists could not conceive of a molecule being able to penetrate a solid body unless the solid body was porous. In a previous talk, Stokes had said that the process by which cathode rays left the outside of the window was similar to that of a billiard ball hitting a row of billiard balls in contact. The ball which leaves the far end of the row is not the same ball as that which hits the row. In his discussion with Lenard, Stokes used the example of the electrolysis of copper sulfate between copper electrodes. If a third electrode of copper is interposed between the two electrodes, copper ions are deposited on one side of the third electrode and removed from the other. But the removed ions are not the same ions as the deposited ions. Stokes mentioned the mechanical effect of cathode rays hitting a vane of a windmill in a Crookes tube. This had the Lenard school stumped.

Certain German physicists (but not Lenard) had replied that there was no more connection between the stream of matter in a Crookes tube and the faint blue light (the cathode rays) than that between the path of a projectile fired from a gun and the sound of the explosion which travels in air. At the end of 1896 both sides were stumped.

On December 17, at a meeting of the Cambridge Philosophical Society, Stokes stated that a stream of charged molecules, on hitting a target in a Crookes tube, gives birth to x-rays. Each charged molecule when it is stopped gives rise to a pulse in the ether. Stokes stated that the x-rays were either nonperiodic or only slightly periodic. Here Stokes had put forward a most valuable idea; but, before it could be properly used, another discovery was necessary. The electron was still unknown at the end of 1896!

The Probability Distribution around a Fix in Celestial Navigation

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BY measuring with a sextant the altitude of a star of known right ascension and declination, a navigator is able to determine a small circle on which he must be situated. Ordinarily the navigator will know his approximate position by independent means, so that, in practice, he is concerned only with so small an arc of the small circle that he may represent it on his chart as a straight line. St. Hilaire's method is commonly used for obtaining the line of position from an observed altitude. In this method, the navigator calculates with the aid of suitable tables the altitude and azimuth which the observed star has at an appropriate assumed position and at the time of the observation. The difference in minutes of arc between the calculated and the measured altitudes gives the distance in nautical miles from the assumed position to the line of position on which the navigator lies; the direction in which this distance must be laid off is determined by the azimuth of the star.

In most cases the assumed position will be near enough to the true position so that the calculated azimuth of the star will be correct within a degree or two, and the orientation of the line of position is therefore subject to no appreciable accidental errors. On the other hand, an error in the measured altitude of the star will result in the line of position being displaced from its true position, parallel to itself, by an amount in nautical miles equal to the error of the measured altitude in minutes of arc. If it be assumed that the measured altitudes are subject to accidental errors which are distributed according to the Gaussian law, then the probability distribution about a single line of position will be geometrically equivalent to the surface whose intersection with a plane perpendicular to the line of position is a Gaussian curve with maximum at the line of position and with constant dispersion in all sections.

By measuring successively the altitudes of two stars, the navigator can obtain two simultaneous lines of position, which together comprise a two-

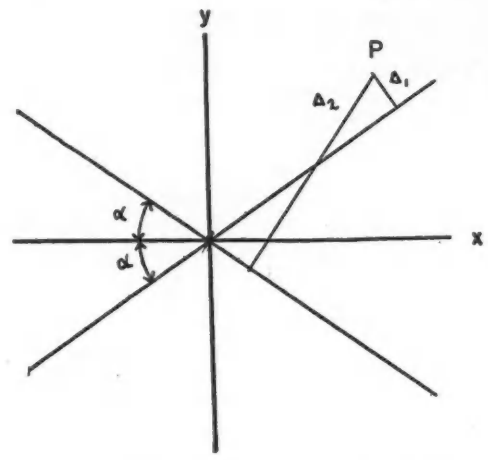

FIG. 1. Two lines of position and their errors.

star fix. Again, the observation of three stars will lead to three lines of position, which comprise a three-star fix. In the case of the two-star fix, it is apparent that the most probable position will be the intersection of the two lines of position; in the case of the three-star fix, the lines of position will form a triangle, and the most probable position will be that at which the sum of the squares of the residuals is a minimum.

It is the purpose of this paper to discover the probability distribution around such fixes; to find how they depend on the azimuths of the stars which are observed; and to compare the probability distribution around a two-star fix with that around a three-star fix. It is assumed throughout that the errors of observation are distributed according to the Gaussian law, and with a constant dispersion which does not depend on the azimuth. The second assumption is likely to be unjustified for observations made with the bubble sextant in flight, and the results of this paper may therefore require modification before being applicable to aerial navigation.

We shall first consider the two-star fix. The probability $p\mathrm{d}\Delta_1\mathrm{d}\Delta_2$ that an observer at P in Fig. 1 will obtain lines of position with linear errors $\Delta_1 \pm \frac{1}{2}\mathrm{d}\Delta_1$ and $\Delta_2 \pm \frac{1}{2}\mathrm{d}\Delta_2$, respectively, is

$$p\mathrm{d}\Delta_1\mathrm{d}\Delta_2 = \frac{1}{2\pi} \exp\left[-\tfrac{1}{2}(\Delta_1^2+\Delta_2^2)\right]\mathrm{d}\Delta_1\mathrm{d}\Delta_2,$$

where the unit of length is the standard deviation σ. Conversely, if the intersection of the two lines of position is taken as the origin of Cartesian coordinates, the x axis being the bisector of the smaller angle 2α between the lines of position, then the probability $p(x, y)\mathrm{d}x\mathrm{d}y$ that the observer is in the elementary area $\mathrm{d}x\mathrm{d}y$ at P is

$$p\mathrm{d}x\mathrm{d}y = \frac{1}{2\pi} \exp\left[-\tfrac{1}{2}(\Delta_1^2+\Delta_2^2)\right]\left|\frac{\partial(\Delta_1, \Delta_2)}{\partial(x, y)}\right|\mathrm{d}x\mathrm{d}y.$$

But

$$\Delta_1 = -x \sin\alpha + y\cos\alpha,$$
$$\Delta_2 = x \sin\alpha + y\cos\alpha,$$

so that

$$p\mathrm{d}x\mathrm{d}y = \frac{\sin 2\alpha}{2\pi} \exp\left[-(x^2\sin^2\alpha + y^2\cos^2\alpha)\right]\mathrm{d}x\mathrm{d}y.$$

The curves of constant probability are therefore the concentric ellipses

$$x^2/a^2 + y^2/b^2 = 1, \qquad (1)$$

where the semiaxes are

$$a = k/\sin\alpha, \quad b = k/\cos\alpha,$$

and the common eccentricity is $\epsilon = (1-\tan^2\alpha)^{\frac{1}{2}}$.

The various contours given by Eq. (1) are distinguished by the value of the parameter k. This parameter can be expressed in terms of the probability P that the observer lies within the corresponding ellipse. We have

$$P = \frac{2\sin 2\alpha}{\pi}\int_0^a \int_0^{b(1-x^2/a^2)^{\frac{1}{2}}} \times \exp\left[-(x^2\sin^2\alpha + y^2\cos^2\alpha)\right]\mathrm{d}y\mathrm{d}x,$$

which readily transforms to

$$P = \frac{4}{\pi}\int_0^k \int_0^{(k^2-u^2)^{\frac{1}{2}}} \exp\left[-(u^2+v^2)\right]\mathrm{d}v\mathrm{d}u.$$

Differentiating with respect to the parameter, we find that

$$\mathrm{d}P/\mathrm{d}k = 2k\exp(-k^2),$$

whence

$$P = 1-\exp(-k^2).$$

The semiaxes are therefore given by the relations

$$a = [-\ln(1-P)]^{\frac{1}{2}}/\sin\alpha,$$
$$b = [-\ln(1-P)]^{\frac{1}{2}}/\cos\alpha.$$

We shall now consider the three-star fix. The probability that an observer at the origin O of coordinates in Fig. 2 will obtain the triangle

$P_1P_2P_3$ is

$$\frac{1}{(2\pi)^{\frac{3}{2}}}e^{-\psi}d\Delta_1 d\Delta_2 d\Delta_3,$$

where Δ_1, Δ_2, Δ_3 are the errors of the respective lines of position, and where

$$\psi=\tfrac{1}{2}(\Delta_1^2+\Delta_2^2+\Delta_3^2).$$

We define the centroid P_0 of the triangle $P_1P_2P_3$ as that point at which the distances δ_1, δ_2, δ_3 to the sides of the triangle are in the same ratio as the corresponding sides, so that

$$\delta_1/l_1=\delta_2/l_2=\delta_3/l_3,$$

where l_1, l_2, l_3 are the sides of the triangle. If (X, Y) are the coordinates of P_0, and κ^2 is the area of the triangle, it is then possible to find relations of the form

$$\Delta_i=a_iX+b_iY+c_i\kappa, \quad i=1, 2, 3,$$

where the coefficients a_i, b_i, c_i depend only on the angles α_1, α_2, α_3 of the triangle and on the orientation of the axes. We may therefore write for the probability that the observer at O will obtain a triangle with the square root of its area in the interval $\kappa\pm\frac{1}{2}d\kappa$, and with its centroid in the elementary area $dX\,dY$ at (X, Y),

$$\frac{1}{(2\pi)^{\frac{3}{2}}}e^{-\psi}\left|\frac{\partial(\Delta_1, \Delta_2, \Delta_3)}{\partial(X, Y, \kappa)}\right|dX\,dY\,d\kappa,$$

where ψ is now given in terms of X, Y, κ. The probability of the occurrence of a triangle of *any* area, with its centroid at (X, Y), is then

$$p(X, Y)dX\,dY=\frac{dX\,dY}{(2\pi)^{\frac{3}{2}}}\times\left|\frac{\partial(\Delta_1, \Delta_2, \Delta_3)}{\partial(X, Y, \kappa)}\right|\int_0^\infty e^{-\psi}d\kappa. \quad (2)$$

Conversely, this must also give the probability that the observer is at the origin of (X, Y) when he obtains a fix with centroid at (X, Y).

To obtain the probability distribution around a given triangle, let the origin of axes (x, y) be taken at the centroid P_0, and let the x-axis make an angle $0\leq\varphi\leq\pi/2$ with the side l_3. Let (x, y) be the coordinates of O relative to the new axes. Then, according to Eq. (2), the probability that the observer lies within the elementary area $dx\,dy$ at O is

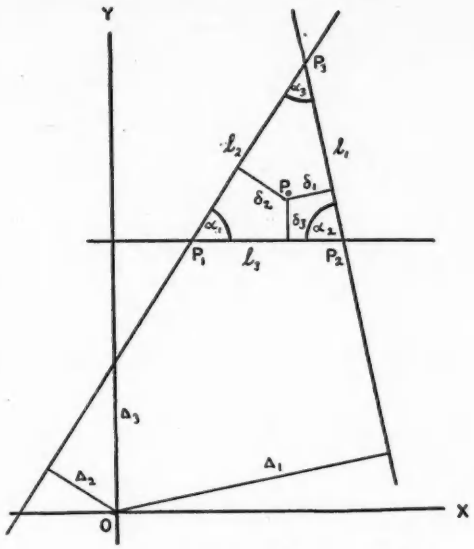

FIG. 2. Three lines of position, their errors and their centroid.

$$p(x, y)dx\,dy=\frac{dx\,dy}{(2\pi)^{\frac{3}{2}}}\left|\frac{\partial(\Delta_1, \Delta_2, \Delta_3)}{\partial(X, Y, \kappa)}\right|\int_0^\infty e^{-\psi}d\kappa$$
$$=\frac{dx\,dy}{(2\pi)^{\frac{3}{2}}}\left|\frac{\partial(\Delta_1, \Delta_2, \Delta_3)}{\partial(x, y, \kappa)}\right|\int_0^\infty e^{-\psi}d\kappa,$$

where ψ is now given in terms of x, y, κ. If we now choose φ so that

$$\tan 2\varphi=\frac{\sin 2\alpha_1-\sin 2\alpha_2}{\cos 2\alpha_1+\cos 2\alpha_2+1},$$

we find that

$$\psi=l^2x^2+m^2y^2+n^2\kappa^2,$$

where

$$l^2=\tfrac{3}{4}-\tfrac{1}{4}[8\cos(\alpha_1+\alpha_2)\cos\alpha_1\cos\alpha_2+1]^{\frac{1}{2}},$$
$$m^2=\tfrac{3}{4}+\tfrac{1}{4}[8\cos(\alpha_1+\alpha_2)\cos\alpha_1\cos\alpha_2+1]^{\frac{1}{2}}, \quad (3)$$

and n^2 is a function of α_1 and α_2. Therefore

$$p(x, y)dx\,dy=\frac{dx\,dy}{(2\pi)^{\frac{3}{2}}}\exp[-(l^2x^2+m^2y^2)]\times\int_0^\infty\left|\frac{\partial(\Delta_1, \Delta_2, \Delta_3)}{\partial(x, y, \kappa)}\right|\exp(-n^2\kappa^2)d\kappa. \quad (4)$$

The integral is a function only of the angles α_1, α_2 and may be regarded as a constant, say I, in an integration with respect to x and y. From

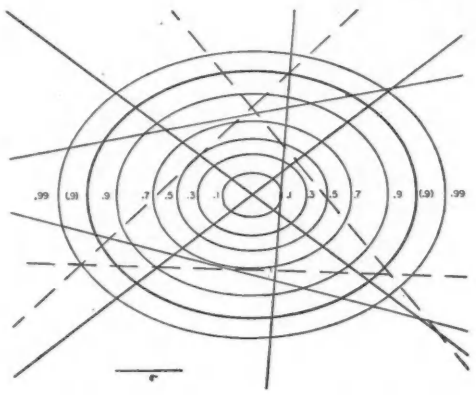

FIG. 3. The heavy lines represent a two-star fix. The heavy ellipse is a curve of constant probability appropriate to this fix; its dimensions are such that the probability is 0.9 that the navigator is within it. The other ellipses are the curves of constant probability appropriate to the three-star fix shown in dashed lines and also to the other three-star fix shown. Each ellipse is marked with the probability that the navigator is within it. The standard deviation σ of a single line of position is the same for all three fixes and has the value shown. Each three-star fix has been drawn so that the root-mean-square deviation at the most probable position is equal to σ.

the condition

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x, y)\mathrm{d}x\mathrm{d}y = 1,$$

it then follows that

$$I = 2^{\frac{1}{2}}\pi^{\frac{1}{2}}lm,$$

so that Eq. (4) becomes

$$p(x, y)\mathrm{d}x\mathrm{d}y = \frac{lm}{\pi}\exp\left[-(l^2x^2+m^2y^2)\right]\mathrm{d}x\mathrm{d}y. \quad (5)$$

As in the case of the two-star fix, the curves of constant probability are ellipses, this time concentric at P_0, and with semiaxes

$$a = k/l, \quad b = k/m. \quad (6)$$

As before, the parameter k can be related to the probability P that the observer lies within the corresponding ellipse, so that, by Eqs. (5) and (6),

$$a = \frac{[-\ln(1-P)]^{\frac{1}{2}}}{l}, \quad b = \frac{[-\ln(1-P)]^{\frac{1}{2}}}{m}.$$

The centroid P_0, which we have taken as the origin of coordinates, is the most probable position; there, the sum of the squares of the residuals is a minimum. By straightforward means, it is found that the residuals δ_i at P_0 satisfy the equation

$$\delta_1^2+\delta_2^2+\delta_3^2 = \frac{2\sin\alpha_1\sin\alpha_2\sin\alpha_3}{\sin^2\alpha_1+\sin^2\alpha_2+\sin^2\alpha_3}\kappa^2,$$

which also gives the internal dispersion of the lines of position. In Fig. 3, the areas of the triangles drawn have been determined by this relation, so that the internal dispersion is unity. However, the scale of the probability distribution is actually independent of the internal dispersion of the lines of position, and therefore it remains the same for all similar triangles, of whatever area.

As indexes of the quality of a celestial fix, three parameters suggest themselves. These are, respectively, the area of the probability ellipse, expressed in appropriate units; the average error r_{Av} of the fix, as defined in the relation

$$r_{Av} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} rp(x, y)\mathrm{d}x\mathrm{d}y;$$

and the probable error R of the fix, as defined in

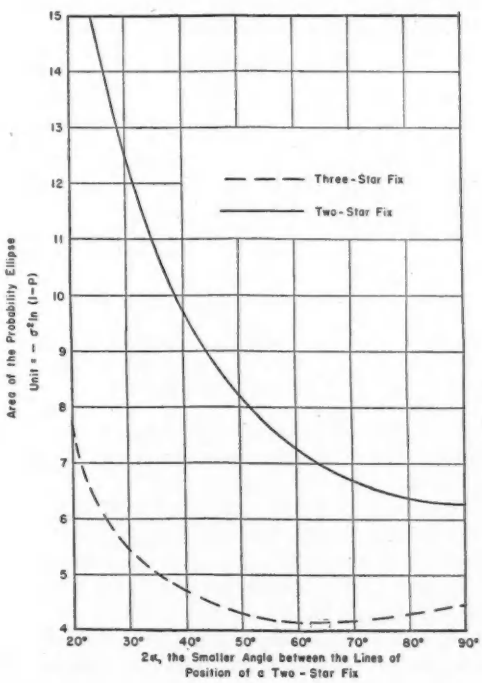

FIG. 4. Area of the probability ellipse around a fix.

TABLE I. The parameters of circular probability distributions.

	Two-star fix (lines of position intersecting at 90°)	Three-star fix (lines of position intersecting at 60°)	Three-star fix / Two-star fix
Area of probability ellipse; unit, $-\sigma^2 \ln (1-P)$	$2\pi = 6.283$	$4\pi/3 = 4.189$	$2/3 = 0.6667$
Average error of fix; unit, σ	$\sqrt{(\pi/2)} = 1.254$	$\sqrt{(\pi/3)} = 1.023$	$\sqrt{(2/3)} = 0.8167$
Probable error of fix; unit, σ	$\sqrt{\ln 4} = 1.180$	$\sqrt{[(2/3)\ln 4]} = 0.965$	$\sqrt{(2/3)} = 0.8167$

the relation

$$4\int_0^R \int_0^{\pi/2} p(x, y) r \, dr \, d\vartheta = \tfrac{1}{2}.$$

The area A enclosed within the curve of constant probability P around a two-star fix is given by

$$A = -\frac{2\pi}{\sin 2\alpha} \ln (1-P).$$

In the case of a three-star fix, the corresponding quantity is conveniently written in terms of the eccentricity as

$$A = -\frac{2\pi(2-\epsilon^2)}{3(1-\epsilon^2)^{\frac{1}{2}}} \ln (1-P);$$

ϵ is readily found from Eqs. (3) and (5) to be

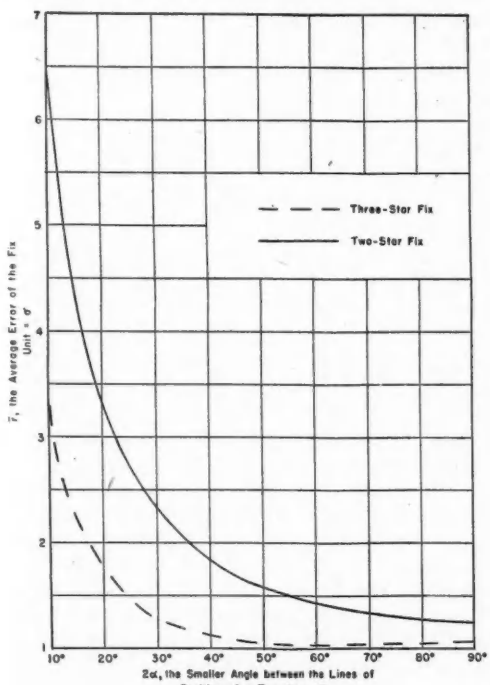

FIG. 5. Average error of a fix.

given by the expression

$$\epsilon = \left[2 - \frac{6}{3+[8\cos(\alpha_1+\alpha_2)\cos\alpha_1\cos\alpha_2+1]^{\frac{1}{2}}}\right]^{\frac{1}{2}}.$$

The average error in the case of a two-star fix is found to be

$$r_{Av} = \frac{E(\epsilon)}{\sqrt{\pi}\sin\alpha},$$

where E is the complete elliptic integral of the second kind. For a three-star fix, we find that

$$r_{Av} = \left[\frac{2(2-\epsilon^2)}{3\pi(1-\epsilon^2)}\right]^{\frac{1}{2}} E(\epsilon).$$

The probable error is not easily expressed in terms of elementary functions except in the cases of zero eccentricity.

As might have been expected, the parameters A, r_{Av}, and R all take their minimum values when ϵ is zero; that is, when $\alpha = 45°$ in the two-star case, and when $\alpha_1 = \alpha_2 = \alpha_3 = 60°$ in the three-star case. It is of interest to compare the parameters of two-star and three-star fixes for these most favorable cases (Table I). It appears that in the most favorable three-star fix the areas are reduced about 30 percent, and r_{Av} and R about 20 percent, below their values in the most favorable two-star fix.

For a further comparison between two-star and three-star fixes, Figs. 4 and 5 have been prepared as follows. It is assumed that a navigator is able to observe three stars, of which he selects for a two-star fix those two whose lines of position intersect most nearly at right angles. As a function of the angle 2α between these lines of position, the figures give, respectively, the area of the probability ellipse in units of $-\sigma^2 \ln (1-P)$, and the average error r_{Av} of the fix in units of σ. The dashed curve in each figure gives the least value that the parameter can assume if all three stars are used. The gain in quality of the three-star fix relative to the two-star fix is seen to be especially important when the lines of position meet at small angles.

Experiments with UHF Wave Guides

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THE subject of ultrahigh frequencies has become of such tremendous importance today that it must be included in every college physics curriculum. It is the purpose of this paper to discuss UHF apparatus and wave guide technics that are suitable for demonstration purposes and also for numerous experimental measurements in both undergraduate and graduate physics courses. First, apparatus will be described for producing and detecting 20-cm and 10-cm microwaves. Then various demonstrations and measurements which can be made with the apparatus will be listed.

20-cm Wave Guide Oscillator

Twenty centimeters was selected as a suitable wavelength because of the availability of the Western Electric 703A ultrahigh frequency triode, which will operate as an oscillator in the 1500-megacycle/sec region. The operating frequency determines the size of wave guide used, and at $\lambda = 20$ cm the minimum diameter circular wave guide which can be used for propagating the TE_{11} wave is 4.6 in. Consequently a wave guide of internal diameter 5 in. was chosen, which has a cutoff wavelength of 21.8 cm. For tubing of this diameter, galvanized-iron stovepipe is found to be satisfactory. It can be made up to an internal diameter of 5 in. within $\pm 1/64$ in. At $\lambda = 20$ cm the wavelength in 5-in. galvanized stovepipe is 50 cm and the attenuation is 0.036 db/m.

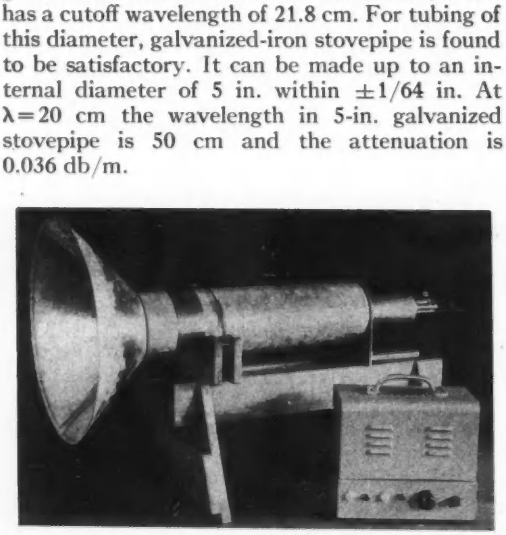

FIG. 1. Wave guide oscillator for generating 20-cm TE_{11} microwaves.

A photograph of the 20-cm wave guide oscillator for generating the TE_{11} wave is shown in Fig. 1. It consists of a 15-in. length of 5-in. ID galvanized-iron stovepipe, to which a horn antenna is attached. The other end of the 5-in. stovepipe is fastened with screws to the oscillator proper. This contains a Western Electric 703A tube mounted on coaxial tuners. The entire oscillator is mounted on a wooden stand with clamps which allow the oscillator to be moved along it. Beside the stand is the power supply for the oscillator.

The principle upon which this oscillator is based was developed by G. C. Southworth[1] and his colleagues at the Bell Telephone Laboratories, for the production of second-harmonic 10-cm waves. In this type of oscillator, a suitable tube is mounted on coaxial tuners inside of a wave guide in such a way that the plate and grid leads of the vacuum tube, together with a portion of the inner conductors of the coaxial tuners, act as a resonant Lecher wire circuit. The electric waves are then radiated from the Lecher wires and propagated along the wave guide.

A schematic drawing of the oscillator, utilizing the 703A triode, is shown in Fig. 2. In the upper part of Fig. 2, the 703A tube is shown mounted on coaxial tuners indicated by the lines numbered *1*, *2* and *3*.

The coaxial tuners, 8 in. long, are made of $\frac{7}{16}$-in. OD, $\frac{3}{8}$-in. ID brass tubing for the outer conductor. The inner conductor is made of $\frac{1}{8}$-in. brass rod and is supported inside the outer conductor by means of a thin polystyrene disk at one end and a brass plug at the other. Tuning of this coaxial line is achieved by means of a metal slider connecting the inner and outer conductors. The position of this slider along the coaxial can be adjusted by means of a wire attached to the slider and running out through the brass plug at the end of the coaxial line. The details of one of the coaxial tuners are shown in the bottom drawing of Fig. 2.

The inner conductors of three coaxial tuners are fastened to the grid and filament prongs of the 703A tube. The outer conductors are fastened to

[1] G. C. Southworth, Proc. IRE. **31**, 319 (1943).

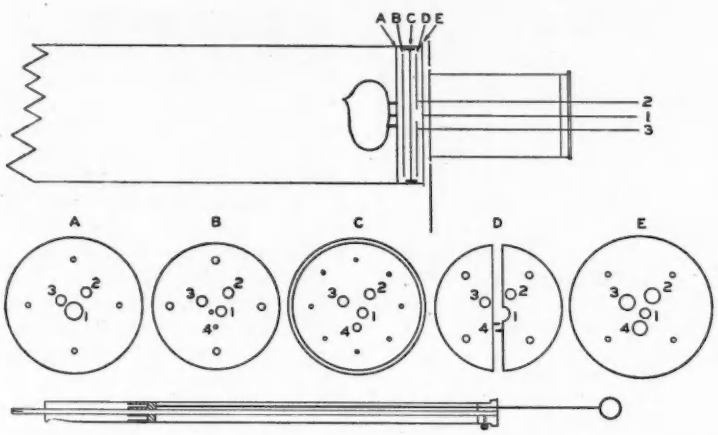

FIG. 2. Schematic drawing of 20-cm wave guide oscillator together with its component parts.

large brass plates which serve as by-pass condensers to prevent radiation coming out on the power line. These brass plates are shown in Fig. 2, and are labelled *A*, *B*, *C*, *D* and *E*. The coaxial line connecting to the grid is numbered *1*, and the outer conductor is soldered to plate *E*, in the hole marked *1*. The inner conductor passes on through hole *1* of the remaining plates, and connects to the grid lead of the 703A. The outer conductors of the coaxial lines *2* and *3* are soldered to plate *D* in the holes *2* and *3*. The outer conductors pass through clearance holes *2* and *3* in plate *E*, and the inner conductors continue through holes *2* and *3* of the remaining plates and are then attached to the filament prongs of the 703A tube by means of short crosspieces. The plate connection to the 703A consists of a brass stud $\frac{3}{16}$-in. in diameter attached to plate *B* adjacent to hole *1*, and extending through the large clearance hole *1* in plate *A*.

Plates *A*, *B*, *C* and *E* are made of $\frac{1}{8}$-in. brass, and plate *D* consists of two semicircular pieces each made of $\frac{1}{16}$-in. brass. These plates are insulated from each other with 0.005-in. mica and are fastened together with screws going into plate *C*. The remaining holes in the plates *A* to *E* indicate screw and clearance holes. Plate *C*, which serves as a shield between plate and filament, has a brass ring around it 0.3 in. wide and $\frac{3}{16}$ in. thick. The total diameter of plate *C*, including the ring, is 5 in. Plates *B* and *D*, $4\frac{3}{8}$ in. in diameter, are set inside the ring around plate *C*. Plate *A* is 5 in. in diameter, and plate *E*, $5\frac{1}{4}$ in. When assembled, 5-in. ID stovepipe can be slipped over all plates, and butted against plate *E*. The stovepipe is then fastened with screws to the ring around plate *C*. It is necessary that the wave guide fit tightly around these plates for proper operation of the oscillator. To insure a pure TE_{11} wave, a wave filter consisting of six parallel wires spaced $\frac{3}{4}$ in. apart should be placed inside the wave guide. These wires are mounted on a ring which fits inside the wave guide, so that the wave filter can be rotated and moved along the guide. Also, it is recommended that the coaxial tuners and condenser plates be either silver plated or made of silver to provide high electric conductivity. Brass is perfectly suitable when clean, but it corrodes easily.

One of the more critical dimensions is the distance of the 703A tube from plate *A*. It was found experimentally that the optimum length of the plate stud and of the inner conductor of the coaxial grid tuner extending beyond plate *A* was $\frac{9}{16}$ in. The plate stud and inner conductor of the grid tuner should be spaced so that the plate and grid prongs of the 703A can slip freely into holes in their ends in which the prongs are fastened by means of setscrews.

The power leads are brought out through hole *4* of plates *B* to *E*. Connections are made to the filament on the two semicircular plates *D*, and the plate lead from plate *B*. The 703A tube is operated with 1.1 v on the filament and 300 v (40–50 ma) on the plate. The power connections to the oscillator are shown in Fig. 3.

The oscillator, with the 5-in. wave guide removed, is shown in Fig. 4. It will be noted that the oscillator is supported by means of a brass plate connected to a clamp, which is fastened to

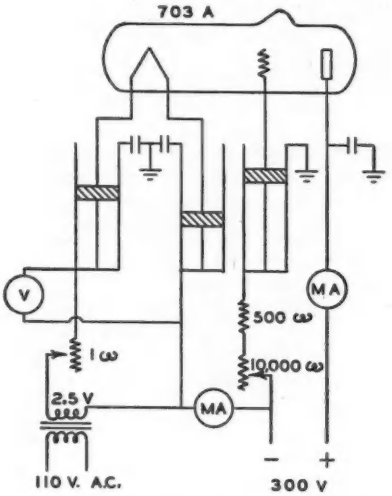

FIG. 3. Circuit diagram of 20-cm wave guide oscillator.

the wooden stand. It is possible to rotate the entire oscillator in this support. The coaxial tuners are supported by a 3-in. brass tube, 5 in. long, screwed to the back of the oscillator, in the end of which is fastened a hard-rubber disk with holes through which the coaxial tuners extend.

To begin operation of the 20-cm oscillator, the coaxial line sliders should be pulled almost all the way out. Oscillation is indicated by the presence of grid current. With the coaxial sliders far out, the frequency of oscillation will, in general, be below cutoff. Next, the coaxial sliders are gradually pushed in toward the 703A tube, thus shortening the length of coaxial line. Finally, a sufficiently high frequency will be obtained, such that radiation will be transmitted from the open end of the 5-in. wave guide. The existence of such radiation can be easily observed by moving a sheet of copper to and fro in front of the open end of the wave guide. If the grid current increases and decreases with this motion, then radiation is being propagated along the guide and out its open end. In general, grid currents will be around 3 to 4 ma with plate currents of 40 to 50 ma and a filament voltage of 1.1 v. The radiofrequency power at $\lambda = 20$ cm is about $\frac{1}{2}$ w.

The frequency range which can be obtained with this oscillator varies from 1400 to 1600 megacycle/sec, corresponding to a wavelength range of 21.4 to 18.7 cm. Since the cutoff wavelength of the 5-in. wave guide is 21.8 cm, wavelengths greater than this cannot be obtained. The lower wavelength end is determined primarily by the 703A tube, which will not deliver much power above 1600 megacycle/sec. Of course, longer wavelengths can be obtained by using pipe of larger diameter. For wavelengths in the region of 25 cm, 6-in. stovepipe has been found suitable.

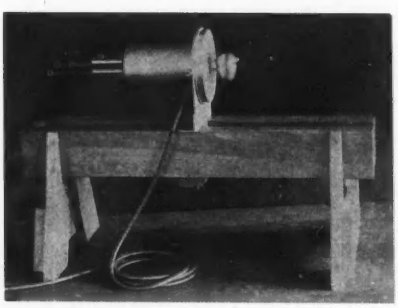

FIG. 4. Twenty-centimeter oscillator with wave guide removed, showing mounting of Western Electric 703A tube.

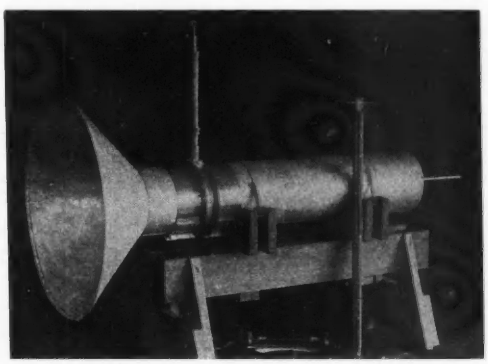

FIG. 5. Twenty-centimeter dipole and wave guide receivers.

20-cm Receivers

Two types of 20-cm receivers are shown in Fig. 5. One consists of a half-wave dipole connected to a crystal detector, and the other is a wave guide receiver which also contains a crystal detector. Western Electric 1N21 crystal detectors were used. However, crystal detectors of the "fixed cat-whisker" type can be made.[2] In the dipole receiver, the crystal detector is mounted

[2] Sarbacher and Edison, *Hyper and ultrahigh frequency engineering* (Wiley, 1943), p. 294.

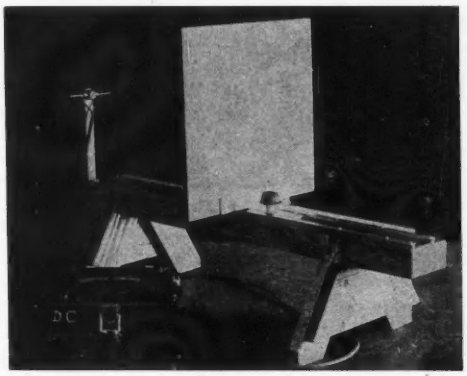

FIG. 6. Wavemeter for measuring free-space wavelengths in 20-cm region.

inside a stick of wood, and two heavy copper wires about 4 cm long are attached to the two ends of the crystal detector. Two-conductor lamp cord is then run from the crystal detector to a milliammeter having current ranges from 0.1 to 10 ma full scale.

The wave guide receiver consists of a section of 5-in. ID stovepipe 18 in. long, in the end of which a piston is mounted. This piston can be moved back and forth along the wave guide by means of a rack and pinion. The rack has a centimeter scale engraved on it so that measurements of the position of the piston can be made. The total travel of the piston is 40 cm. The piston has brass fingers around its edge which make contact with the inside of the wave guide. Attached to this section of the guide is a 3-in. length of 5-in. pipe. This latter section of pipe contains a crystal detector and coaxial tuner. The coaxial tuner is fastened into a brass block attached to the 3-in. section of guide and is insulated from the brass block with 0.015-in. thick polystyrene. This serves as a by-pass condenser across which connections are made for measuring the crystal current. The crystal detector is mounted at the center of the wave guide on an extension of the inner conductor of the coaxial line. This inner conductor is continued from the other side of the crystal detector, and is attached to the wave guide on the side opposite to that to which the coaxial line is attached. Brass flanges are soldered to the outside of this 3-in. section of stovepipe to give it added strength. This part of the receiver is then clamped to the section of guide containing the piston.

Antennas

Clamped to the 20-cm wave guide oscillator and receiver are horn antennas. These horns, designed from data by Southworth and King,[3] have a gain of 13 db. Each is formed from a cone of diameter 2λ, or about 16 in., and of total axial length λ. The angle of flare is then 90°. These horns have an angular beam width of 32° at the half-power points. Higher-gain horns of greater directivity can be made by increasing the diameter and axial length and, in general, decreasing the angle of flare.

Wavemeters

Two types of wavemeters are desirable for experimental work on wave guides: one for measuring wavelengths in free space; and another for measuring the wavelength inside the guide, and the reflection coefficients of devices connected to, or placed inside, wave guides.

A simple wavemeter for measuring wavelengths in free space is shown in Fig. 6. It consists of a half-wave dipole fastened to a crystal detector which is mounted in a hard-rubber block. The hard-rubber block can be rotated in its support so that the dipole will be parallel with the lines of electric intensity. On the stand is mounted a board 1 ft square, faced with a sheet of copper. The board can be moved to and from the dipole in ways by means of a rack and pinion. Measurements of the position of the board are made with a meter stick along the stand. The microwaves

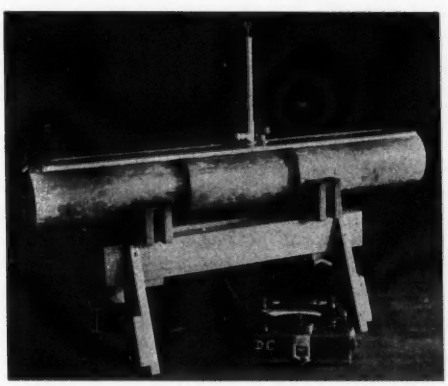

FIG. 7. Twenty-centimeter standing-wavemeter.

[3] G. C. Southworth and A. P. King, Proc. I.R.E. **27**, 95 (1939).

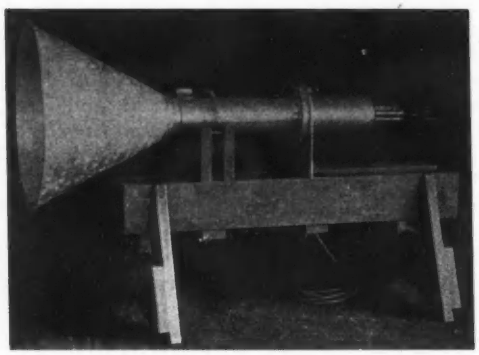

FIG. 8. Wave guide oscillator for generating second-harmonic, 10-cm TE_{11} microwaves.

are reflected from the copper sheet, thus producing standing waves in air. Maxima or minima of the standing waves are indicated by the crystal detector current. The distance between successive minima or maxima is $\frac{1}{2}\lambda$. With this wavemeter it is possible to measure wavelengths to ± 0.5 mm, provided the copper reflector is approximately at right angles to the direction of propagation of the microwaves. This wavemeter is satisfactory for almost all wavelengths in the 20-cm region, since the tuning of the dipole is not sharp.

The standing-wavemeter (Fig. 7) consists of a 3-ft length of 5-in. ID stovepipe, along the length of which a narrow slot is cut. Into this slot the inner conductor of a coaxial line extends $\frac{1}{4}$ to $\frac{1}{2}$ in. A crystal detector is connected across the inner and outer conductor of the coaxial line, and the entire assembly can be moved in ways along the slot by means of a rack and pinion. The upper part of the outer conductor of the coaxial line is insulated from the lower part (which consists of a block of brass containing the crystal detector) with 0.015-in. thick polystyrene which serves as a by-pass condenser. Connections for measuring crystal current are across this by-pass condenser. When the coaxial line is properly tuned, crystal currents of several milliamperes are obtained. Standing waves are detected by moving the coaxial line along the slot in the wave guide and measuring maximum and minimum crystal currents. The standing-wave ratio is given by the square root of the ratio of maximum to minimum crystal currents. By measuring the positions of the coaxial line at minima or maxima crystal currents, half wavelengths inside the wave guide are determined. Wavelengths inside the guide can be measured to within ± 1 mm.

10-cm Oscillator and Receiver

The 20-cm oscillator can be converted into a second-harmonic 10-cm oscillator by making use of the cutoff property of wave guides. If a 3-in. OD, $2\frac{7}{8}$-in. ID brass pipe is clamped to the oscillator proper, the 20-cm waves will not be propagated along it; instead, second-harmonic TE_{11} 10-cm waves will be produced. The oscillator arranged for 10-cm waves is shown in Fig. 8. In Fig. 9 is shown the wave guide receiver for 10-cm waves. It is similar in construction to the 20-cm wave guide receiver, except that 3-in. OD, $2\frac{7}{8}$-in. ID brass tubing 12 in. long is used, and the length of the coaxial tuner is 4 in. instead of 8. The latter is mounted in a $1\frac{1}{2}$-in. section of 3-in. tubing which is clamped to the 12-in. section containing the piston. Attached to the 3-in. wave guide of the oscillator and receiver are two horn antennas.[3] These horns have a gain of 17 db and an angular beam width of 21° at the half-power points. They are 3λ in diameter and have a total axial length of 3λ.

The energy available at 10 cm, using the second harmonic, is between 2 and 3 mw, about 1/200 the power available when the oscillator is used at its fundamental frequency. However, sufficient power at 10 cm is available for most demonstration and measurement purposes.

Demonstrations and Measurements

Numerous demonstrations and measurements can be made with the apparatus described. For demonstration purposes, the dipole or wave guide receiver can be connected to an audio amplifier and loud-speaker, so that the 60-cycle/sec modulation, which occurs because of alternating current on the filament of the 703A, can be heard. Below are listed some demonstrations and measurements. The list is by no means complete, and others can be found in the literature.[4]

(1) Using the 20-cm transmitter as shown in Fig. 2, polarization of the radiation can be shown with the dipole receiver. Also the directivity of the horn antenna can be demonstrated qualitatively.

(2) Reflection and absorption of the radiation with a copper sheet can be shown, using the dipole receiver. Standing waves in air can be produced by this means, and an approximate wavelength measurement made. Parallel wires formed by winding wire spaced $\frac{1}{4}$ in. apart

[4] Bell Lab. Rec. 18, 194 (1940); Brainerd, Koehler, Reich and Woodruff, *Ultra-high-frequency techniques* (Van Nostrand, 1942), pp. 516–19.

around a square wooden board can also be used to demonstrate reflection, polarization and absorption, depending upon the orientation of the wires with respect to the electric field. Finally, reflection and absorption of the radiation by screens of various mesh can be compared with that obtained with the copper sheet and parallel wires.

(3) Transmission by wave guides of various diameters can be demonstrated by clamping the dipole receiver 4 ft in front of the transmitter and inserting between transmitter and receiver 3-ft lengths of stovepipe of various diameters. If 6-, 5- and 4-in. ID stovepipes are used, the 6- and 5-in. guides will transmit the microwaves, while the 4-in. guide will not. The latter demonstrates the cutoff property of wave guides very effectively. The transmission by wave guide bends can also be demonstrated by using a standard 5-in. stovepipe elbow to the ends of which sections of 5-in. guide are clamped. For this case the microwaves are guided around a right-angle bend.

(4) If the wave guide receiver is placed at one end of a long table and the transmitter at the other, the directivity of the horn antenna can be demonstrated by rotating the receiver. The sharpness of tuning of the receiver and also the wavelength inside the guide can be demonstrated by moving the piston inside the receiver back and forth. If the receiver is placed alongside the transmitter, the radiation can be reflected from the copper sheet at a considerable distance from transmitter and receiver.

(5) Precise measurements of wavelength can be made in air by using the wavemeter in Fig. 6, and in the guide, by using the wave guide receiver. For such measurements the crystal currents should be recorded with a milliammeter. The wavelength in air *versus* wavelength in the guide can be plotted for the various frequencies obtainable with the 20-cm oscillator and the resulting curve compared with the equation $\lambda_g = \lambda\lambda_c/(\lambda_c^2 - \lambda^2)^{\frac{1}{2}}$, where λ is the wavelength in air, λ_g the wavelength in the guide and $\lambda_c = 1.71d$ the cutoff wavelength for the TE_{11} wave in a guide of internal diameter d.

(6) The radiation pattern in both E and H planes can be obtained by placing the transmitter in one corner of a large room and the receiver with the horn whose pattern is to be measured in the opposite corner; or, preferably, both transmitter and receiver can be placed out of doors where free-space propagation can be

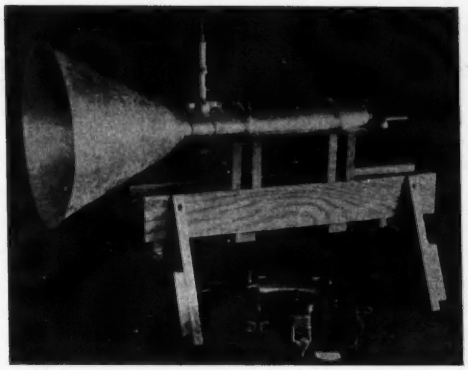

FIG. 9. Ten-centimeter wave guide receiver.

insured. The radiation pattern is obtained by rotating the receiver first in the E plane and then in the H plane and measuring the receiver crystal current as a function of angle. If the crystal current is proportional to the square of the applied voltage, then the current is proportional to the microwave power. If the crystal is not "square law," then a voltage *versus* current calibration should be made. The gain of the antenna can be compared to that of the dipole by comparing the maximum crystal currents of each. The gain of a dipole is 1.5 times the gain of a nondirective antenna, and so the horn gain can be compared to that of a nondirective antenna and hence the "absolute gain" of the horn obtained.

(7) Standing-wave measurements which indicate the smoothness of a wave guide transmission line can be made by inserting the standing-wave-meter (Fig. 7) into the line. Standing waves due to bends in the line, the open end of the wave guide, or a horn termination can be readily measured. The standing wave ratio is given by the square root of maximum to minimum crystal currents, assuming that the crystal is "square law." Measurements on the 13-db horn antenna gave a standing-wave ratio of 1.15 db; on the open end of the 5-in. wave guide, 1.8 db; and on the 5-in. right-angle stovepipe elbow, 3 db in both E and H planes. From these measurements it is evident that the horn antenna is a reasonably good termination of a wave guide transmission line, and also that the stovepipe elbows serve well as bends.

I wish to express my indebtedness to Professor C. A. Proctor for the photographs of the apparatus, and to Mr. W. Durrschmidt and Mr. E. Fitzgerald, who helped construct the apparatus.

A Series of Magnetic Experiments

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EXPERIMENTS in magnetism for undergraduate courses have been rather scarce. The usual magnetic experiments consist in obtaining a hysteresis loop for a toroidal specimen of iron, determining the earth's magnetic field, or obtaining magnetization curves for coils of various types, such as an air-core coil, a straight-bar iron-core coil or a loop iron core with an air gap. The scarcity of experiments in the domain of magnetism may be attributed to two causes: the lack of commercially available equipment for any but the simple experiments, which necessitates building and trying out a variety of equipment; and the fact that the measuring technics must be more elaborate than in the case of many fundamental electric circuit experiments.

In view of the growing importance of magnetism and magnetic circuits, it is desirable to develop a group of experiments to supplement the classroom theory. During the past several years the authors have been developing magnetic experiments for the undergraduates in our Electrical Engineering Course. While the series of experiments has not as yet been completed, many interesting results have been obtained. Some of the first experiments are qualitative in nature but the latter ones are all quantitative.

Magnetic Field Due to Two Parallel Wires

The first of the qualitative experiments is an investigation of the magnetic field about two conductors carrying current. The apparatus consists of two $\frac{3}{16}$-in. brass rods mounted as shown in Fig. 1. Connections to these rods can be made so that the current in one is in the same direction or in an opposite direction to that in the other rod. Link C is removable, and leads a and b can be attached to A and B. A current of about 30 amp in each wire produces a magnetic field strong enough so that the pattern may be observed by sprinkling iron filings. Less current is required if the field pattern is obtained by the use of a small compass. The latter method is probably the more instructive, although it has the disadvantage that the current must be maintained for a considerable time.

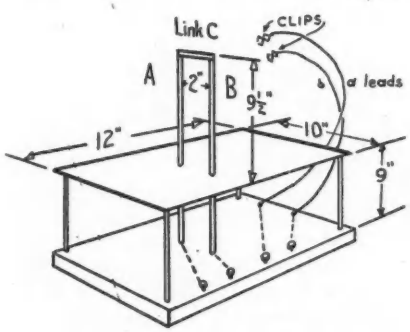

Fig. 1. Apparatus for magnetic field about parallel wires.

Magnetic Field of a Coil

The second experiment consists in obtaining magnetic field patterns of a short double-layer solenoid as shown in Fig. 2. The coil is formed on a circular brass frame of diameter 7 in. and consists of 25 turns of No. 18 copper wire with a tap at ten turns (Fig. 3). It can be used later as a tangent galvanometer coil without any modification, since the top surface of the plotting board has been made to coincide with the center of the coil. In both Figs. 1 and 3, all return leads are mounted far enough from the plane of the board so as not to interfere with the magnetic pattern.

Ballistic Galvanometer as Flux Meter

The first of the quantitative experiments is that of calibrating a ballistic galvanometer as a flux meter. The student will use this galvanometer in all succeeding experiments. The calibration is obtained both by the condenser discharge method and by the use of a long solenoid. Condensers to be used for calibration purposes must be chosen with care or the calibration obtained may be as much as 20 percent lower than that obtained by the solenoid method; some experimental results are shown in Table I. Since a constant voltage E

Table I. Value of constant K for ballistic galvanometer used as flux meter.

Type of calibration	K (maxwell turn/mm)
Long solenoid	1616
W. E. condenser	1609
Condenser which polarized:	
Charging time, 30 sec	1452
Charging time, 60 sec	1307

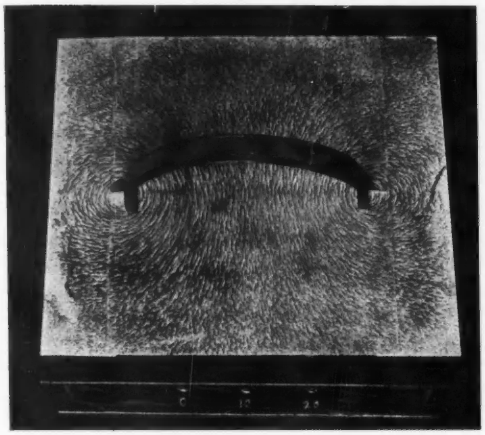

FIG. 2. Magnetic field of a short double-layer solenoid.

is applied to the condenser, the dielectric must be free from serious polarization[1] or the total charge Q of the condenser will not be given by the simple relationship, $Q=CE$. Again, a condenser that is accurate at a certain frequency may deviate considerably from its calibrated value as the frequency is changed.[2] In our work we have found that Western Electric condensers calibrated at 1 kc/sec against a General Radio Standard give the best and most consistent results. The results show no variation with time taken to charge the condenser. The calibration is made with various charges Q, so as to obtain deflections covering the galvanometer scale, and also with a reversed polarity of the battery, so as to use both directions of galvanometer movement.

The long solenoid (Fig. 4) is wound on a fiber tube 54 cm long and 5.71 cm in diameter. The primary coil consists of 1025 turns of No. 20 enameled single cotton-covered copper wire wound in a single layer. The secondary coil is 1.27 cm long, 0.63 cm thick and 6.03 cm in diameter, and has 200 turns wound in layers. It is free to be moved from one end of the primary to the other. In calibrating the galvanometer, the secondary coil is placed at the center of the primary winding and the relationship[3]

$$\phi=4\pi NI\mu A/10L \qquad (1)$$

is used to calculate the flux ϕ (maxwells). Here I (amp) is the current; A (cm²), the cross-sectional area of the coil; N, the number of turns; μ, the permeability; and L (cm), the length of the solenoid. In Table I the constant K is expressed in maxwell turns per millimeter; and the condenser calibration Q/d, expressed in microcoulombs per millimeter, is obtained experimentally and converted by the relationship[4] $K=100rQ/d$, where r (ohms) is the critical damping resistance, d (mm) is the galvanometer deflection caused by passing a charge Q (μcoul) through the galvanometer.

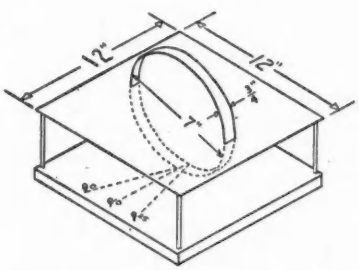

FIG. 3. Apparatus for magnetic field of a coil.

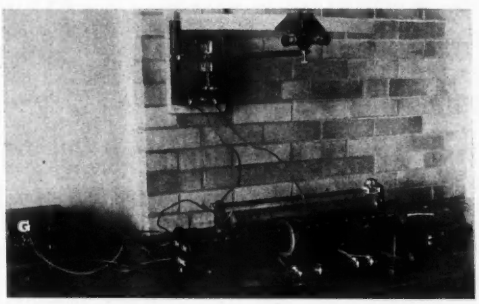

FIG. 4. Long solenoid.

In all cases the galvanometer was in a circuit of resistance equal to its critical damping resistance.

The last two values in Table I are for the same condensers, but the charging times were different. The lowering of the value of K was noted with these condensers even for times of charging up to 8 min.

[1] Manning and Bell, Rev. Mod. Physics 12, 226–230 (1940).

[2] See, for example, the curves for the General Radio Type 380 condenser in *General Radio Catalog K* (ed. 2).

[3] Timbie and Bush, *Principles of electrical engineering* (Wiley), ed. 3, p. 374.

[4] "Notes on moving coil galvanometers, Note Book ED1, 1943" (Leeds and Northrup), pp. 38–39.

FIG. 5. Magnetic circuit used to study leakages and fringing flux patterns.

With the value of K known, the flux of the long solenoid is investigated by moving the secondary coil along the axis of the primary coil. The resulting experimental curve for flux ϕ *versus* distance s (cm) from the midpoint of the coil should be compared with the curve given by the equation[5]

$$\phi=\frac{2\pi NI\mu A}{10L}\left[\frac{\frac{1}{2}L+s}{\sqrt{(r^2+(\frac{1}{2}L+s)^2)}}+\frac{\frac{1}{2}L-s}{\sqrt{(r^2+(\frac{1}{2}L-s)^2)}}\right], \quad (2)$$

where r (cm) is the radius of the solenoid.

Leakage Flux

The magnetic circuit used in this experiment consists of 63.5 cm of 2.54×2.54-cm cold-rolled bar stock SAE 1010, bent into two U-shaped pieces which are joined by a ground lap joint under the lower coil. The iron forms a complete path except for the 0.318-cm air gap located between the two coils and visible in Fig. 5. Each of the coils *1*, *2* and *3* (Fig. 6) has 1500 turns of No. 18 enameled, single cotton-covered wire wound in layers; the over-all length of the coil, including the fiber winding form, is 7.62 cm. Each coil is connected to binding posts which enable the coils to be used separately or in aiding or opposing connections. On the iron core at the positions *1*, *2*, *3*, *4*, ···, *11* are three-turn exploring coils, which are used to find the flux density distribution around the iron path when the exciting ampere turns are supplied by various combinations of the three coils. Figure 7 shows the flux densities at the 11 fixed positions when the coils *1*, *2*, *3* are excited in series aiding. It is to be noted that this distribution shifts as the exciting current is increased. Figure 8 shows the variation of flux density at two fixed positions as the current is increased through the three coils in series aiding. Table II gives the flux density at the various positions when each exciting coil is used separately. Other combinations of exciting coils give various leakage effects. In addition, plugs of 0.318-cm iron, 0.159-cm iron plus 0.159-cm brass, and so on, may be inserted in the air gap to produce still other patterns. When the shorter air gaps are used, it becomes increasingly important to completely demagnetize the core before each reading.[6-9]

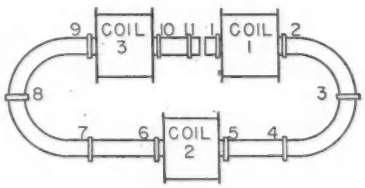

FIG. 6. Positions of search coils in magnetic circuit.

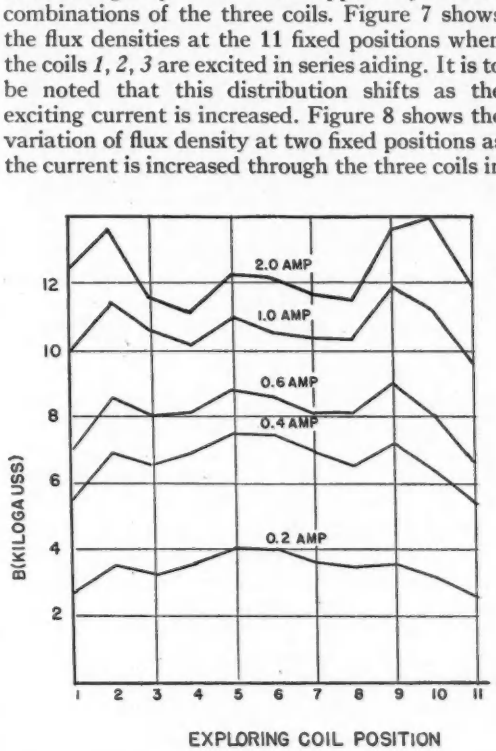

FIG. 7. Distribution of flux density when coils *1*, *2* and *3* (Fig. 6) are connected in series aiding.

[5] Reference 3, p. 369.

[6] A. W. Smith, *Electrical measurements in theory and application* (McGraw-Hill), p. 240.

Hysteresis Loop

The equipment is shown in Fig. 9. The primary, or exciting, coil consists of 2000 turns of No. 18 enameled, single cotton-covered wire. The secondary, or search, coil is wound beneath the exciting coil and consists of 10 turns tapped at 3 and 5 turns, thus providing exploring coils of 2, 3, 5, 7 or 10 turns. The exploring coils are 4.15 cm in diameter, but the difference between this exploring coil and one wound in contact with the iron is negligible until the iron becomes saturated. The iron core is 43 cm long, 36 cm being the mean length of the U-shaped piece and 7 cm the mean magnetic length of the straight piece, or keeper. The contact-making surfaces of the yoke and keeper are ground smooth, thus eliminating the

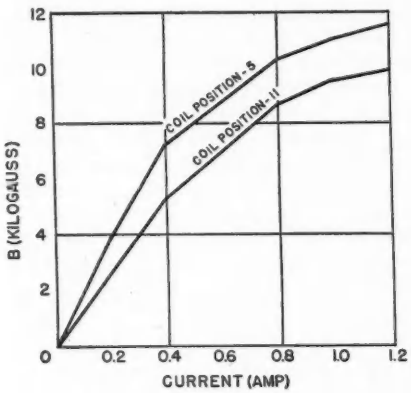

FIG. 8. Flux density *versus* exciting current for position *5* and *11*, Fig. 6.

air gaps associated with uneven surfaces. While the exciting ampere turns are not symmetrically located as in the case of a uniformly wound ring core, a reasonably satisfactory curve is obtained. Figure 10 shows the hysteresis loop obtained with an SAE 1010 cold-rolled core by the usual step-by-step method. The area of the loop is obtained by a planimeter for calculating the energy loss. The advantages of the apparatus are that cores of various types may be inserted and that it may be used for later experiments.

[7] A. W. Smith, "Demagnetization of iron," Physical Rev. 10, 284 (1917).
[8] *Standard handbook for electrical engineers* (McGraw-Hill), ed. 6, sec. 4, p. 387.
[9] S. R. Williams, *Magnetic phenomena* (McGraw-Hill), p. 61.

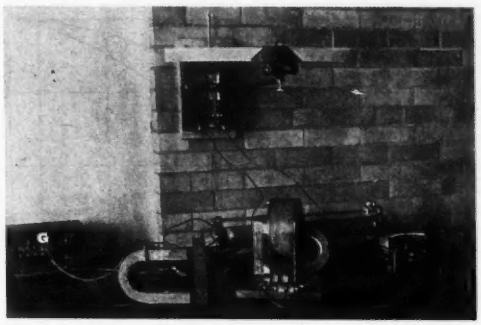

FIG. 9. Excitation coil and core used in hysteresis loop and magnetic circuit experiments.

The Magnetic Circuit

The apparatus is the same as that of the preceding experiment except that the student is supplied with the magnetization curve of the material he is to use as a core. As preliminary preparation, he is asked to draw the curves of flux density *versus* total ampere turns for the following cases:

(1) 43-cm iron core, 2.54×2.54 cm, with no air gap.
(2) 43-cm iron core plus two air gaps of 0.005 cm each.
(3) 43-cm iron core plus two air gaps of 0.010 cm each.
(4) 43-cm iron core plus two air gaps of 0.020 cm each.

In the experimental set-up, the air gaps are obtained by using spacers of 0.005-cm brass placed on top of each other to obtain the required gap. Figure 11 shows typical results for this

TABLE II. Flux density at each of the 11 exploring coils when each of the exciting coils, *1*, *2*, *3*, is excited in turn with 1.2 amp.

Exploring coil position	Flux density (kilogauss) excited by coil		
	1	*2*	*3*
1	7.00	3.20	4.81
2	9.31	4.50	4.90
3	7.53	5.90	4.66
4	6.36	7.60	4.66
5	5.51	9.55	4.97
6	4.64	9.19	5.73
7	4.47	7.43	6.75
8	4.47	8.35	8.00
9	4.81	4.63	10.12
10	5.24	3.63	8.44
11	5.00	3.03	6.34

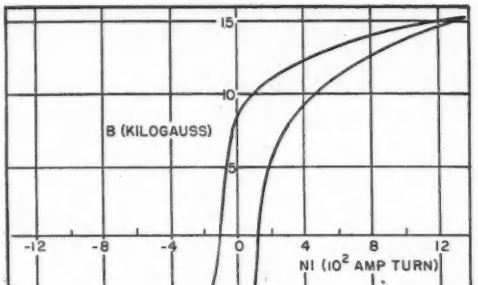

FIG. 10. Hysteresis loop for an SAE cold-rolled core. The residual magnetism is 9.15 kilogauss and coercive force is 1920 ampere turns.

experiment. It is to be noted that the difference between calculated and experimental results increases as the air gap is made larger, owing to leakage paths of flux. A more uniform distribution of exciting turns would tend to eliminate this result. However, the student, having previously performed the experiment on leakage, is in a position to discuss intelligently the difference between the two sets of data. Some of the discrepancies may be eliminated by the usual approximation of increasing the area of the air gap. It is customary to assume that the effective width is equal to the actual width plus the length of the gap and that the breadth is likewise increased.[10] Even with this correction, there is some deviation from calculated results because of the unsymmetrical and nonuniform excitation.

If the experimental set-up is to duplicate the conditions assumed in the calculations, it is necessary that the iron be demagnetized before each reading of the galvanometer. This means that a slow process of simultaneously reducing and reversing the current in the exciting coil must be employed.[6] To investigate the possibility of using alternating current for the demagnetizing current supply and as a basis for a future experiment, the effect of frequency on the demagnetization of the iron core was studied. Cores of SAE 1010 cold-rolled iron bar stock were made of 2.54×2.54-, 1.9×1.9-, 1.27×1.27-, 0.63×0.63-cm material, each core being 36 cm long and having a 7-cm keeper of corresponding material and cross section. The core was demagnetized by reversals of direct current in 5-sec intervals as the current was reduced from 0.5 amp in 50-ma steps until 120,000 ohms had been inserted in series with the 24-v d.c. supply. The iron was found to be demagnetized by this process. The flux associated with a 0.5-amp direct current was then determined experimentally. The residual magnetism of the core was evaluated by finding the intercept of the hysteresis loop with the zero H-axis, that is, by opening the circuit, thus reducing the current from 0.5 amp to zero. This is the residual magnetism with no demagnetization whatsoever. An a.c. supply of definite frequency was then connected to the exciting coil and the current increased until 0.355 amp rms, or 0.502 amp peak, was obtained. The resistance in series with the a.c. supply was increased slowly until the total resistance of 120,000 ohms was inserted. The value of the alternating current in all cases was less than 0.001 amp peak when the demagnetizing supply was opened. The total flux associated with the closing of the exciting coil on a 0.5-amp d.c. supply was then evaluated. This value of flux subtracted from the value of flux obtained by closing the exciting coil on a 0.5-amp d.c. supply after slowly demagnetizing with a d.c. supply is used as a measure of the residual flux after a.c. demagnetization.

The frequency was changed and the test repeated. The a.c. supply for these tests was an

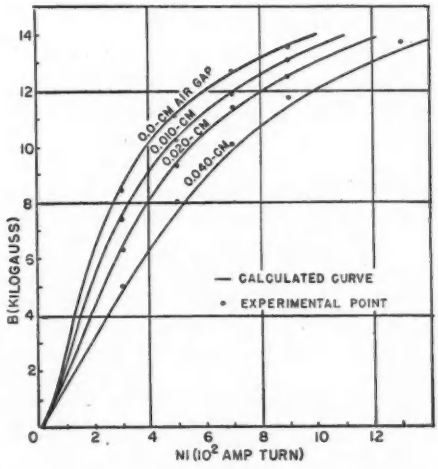

FIG. 11. Typical results for air gaps of various lengths.

[10] Reference 3, pp. 386–388.

alternator driven by a d.c. motor, its speed controlled by adjustable armature voltage. In all cases the use of alternating current resulted in some demagnetization, but the amount depended upon the core size and the frequency. Figure 12 shows the circuit used; Fig. 13, the results obtained. To obtain a high degree of demagnetization for the large (2.54×2.54-cm) core, we have used a motor-driven mechanical switch reversing the d.c. polarity at a rate of one complete cycle per second. This time lag in magnetization has been discussed by several authors[6–8] and has been attributed to the induced currents in the core; consequently it is more noticeable in cores of large cross-sectional area, which have lower resistance to the induced currents. The results clearly indicate the necessity of slow demagnetization of iron cores of large cross-sectional area, if complete demagnetization is to be obtained. Tests to evaluate the effect of alternating current used for demagnetization have shown that its magnitude has little effect as long as the peak value is nearly equal to the d.c. value used to magnetize the core. Longer times of application of the alternating current, until the temperature of the core increased to 40°C, also have no effect on the demagnetization.

Fringing of Flux About an Air Gap

The equipment is the same as that for the leakage flux experiment (Fig. 5). The checkerboard of the holes in the board supporting the magnet is clearly visible in the area inside the iron path and also outside the iron in the vicinity

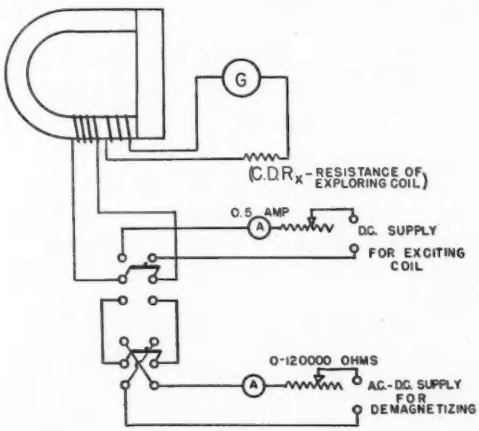

FIG. 12. Circuit for experiments on hysteresis and residual magnetism.

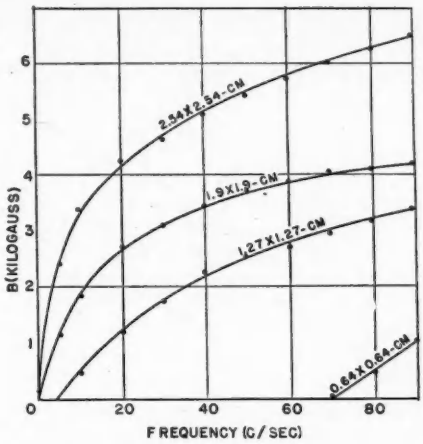

FIG. 13. Effect of frequency of a demagnetizing current on the residual magnetism, for SAE 1010 iron cores of various sizes.

of the air gap. The small search coil is also visible, lying on top of one of the excitation coils.

The search coil consists of 200 turns of No. 39 wire wound so as to make a coil 1.27×1.27 cm and 0.317 cm long. The center of the search coil is directly over the central support, and the height of the support is such that the center of the coil is in the plane formed by the center line of the iron core. The construction of the search coil is shown in Fig. 14. The holes of the checkerboard are 0.635 cm apart between centers in each direction. When the search coil supports are placed in the holes, the center of the coil is directly over one of them and the component of the flux perpendicular to the coil is measured. By rotating the coil through an angle of 90° about the center support, the component at right angles to the first is measured. The two components, omitting the vertical component, allow a plan view of the flux density distribution to be obtained, since the direction and magnitude are known. The points of equal flux density are evident from the resulting graph. This flux density investigation is an indication of the fringing flux. Figure 15 is a graph of the magnetic field around a 0.318-cm air gap when the excitation is supplied by the three coils all producing flux in the same direction and excited with 1 amp. The complete coverage of the area inside the magnetic core with holes, the possibility of other coil connections and different air gaps allow various other patterns of fields to be obtained. All deflections on the galvanometer were obtained

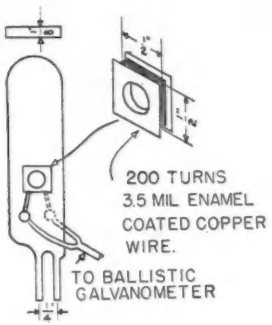

FIG. 14. Construction of search coil.

by abruptly dropping the exciting current from 1.0 amp to zero.

Conclusion

The experiments described have been tried in our courses and have given consistent results for the four duplicate sets of equipment which we have built. As the student becomes more familiar with the procedure of making flux measurements with the ballistic galvanometer, he can obtain more data in a given time, so the fringing flux experiment can be easily performed.

At present we have two additional experiments under construction. The first is the standard Epstein test for core loss.[11] The second is an experiment to obtain a quantitative check of the equation of magnetic pull. Other experiments which have been suggested by the present work are the effect of frequency on demagnetization of various sized cores, and the effect of laminated cores in place of the solid cores used. The present equipment can also be used to illustrate the effect of various cores on inductance and the effect caused by mutual inductance.

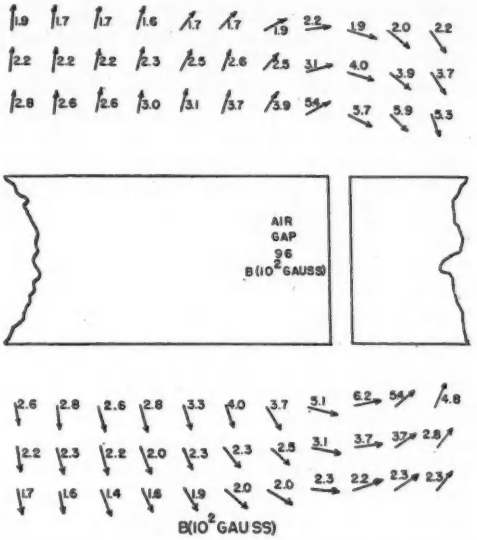

FIG. 15. Fringing effects around the air gap of the magnetic circuit (Figs. 5 and 6). The excitation is not symmetrical about the air gap.

We are indebted to Mr. J. Schulte, who constructed all of the special equipment described, and to Mr. C. R. Egry, of the Mechanical Engineering Department, for the photographs.

[11] The equipment will be built according to specifications in Publication A257-43T, American Society for Testing Materials (260 S. Broad St., Philadelphia, Pa.).

THE most important problem that faces science today is to bring up to date our methods of transmitting and reviewing the mass of research which has accumulated through the years. We have so much knowledge that we are engulfed in a sea of publications in which we flounder. In the age of airplanes we must plow through this in square-rigged ships. We have not employed the inventions that we have to make this easier.—VANNEVAR BUSH.

Reproductions of Prints, Drawings and Paintings of Interest in the History of Physics

23. The First Electric Battery

E. C. WATSON
California Institute of Technology, Pasadena, California

THE world was given for the first time a means of producing a steady electric current by the work of ALESSANDRO VOLTA (1745–1827), the bicentenial of whose birth occurs this year. VOLTA'S work culminated in the famous letter to SIR JOSEPH BANKS, President of the Royal Society, dated March 20, 1800, and published in the *Philosophical Transactions* **90**, 403 (1800), in which he described his electric pile. This paper ushered in the second great period in electrical science in much the same way as GILBERT'S *De magnete* had ushered in the first great period two hundred years earlier.

As SARTON has said, by the discovery of the electric cell mankind was "enriched with a new instrument of untold power. One might compare this discovery with that of the telescope and microscope two centuries earlier, with this essential difference, however, that after all these older instruments were only means of magnifying our vision. They enabled us to see things which we could not see before, but which existed nevertheless. The telescope did not create the stars, nor the microscope, infusoria, except in man's consciousness. On the contrary, the electric cell was really a creative instrument; it opened to man a new and incomparable source of energy."[1]

Plate 1 is reproduced from the paper of 1800 and shows the two earliest forms of the electrolytic battery, the "crown of cups" and the "voltaic pile." Plate 2 is from a photograph of one of the original piles, which unfortunately were destroyed by fire in 1899. Plate 3 shows VOLTA at the age of 40. It is from the portrait by SABATELLI; the engraving was used as a frontispiece to the collection of VOLTA'S work published at Florence in 1816.

[1] G. Sarton, "The discovery of the electric cell," Isis **15**, 127 (1931).

Since VOLTA'S original letter to BANKS was written in French and since copies of the *Philosophical Transactions* for 1800 are available only in a few of the larger libraries, it may be useful to reprint at the end of this article an English translation of the complete paper[2] which appeared in the *Philosophical Magazine* **7**, 289 (1800).

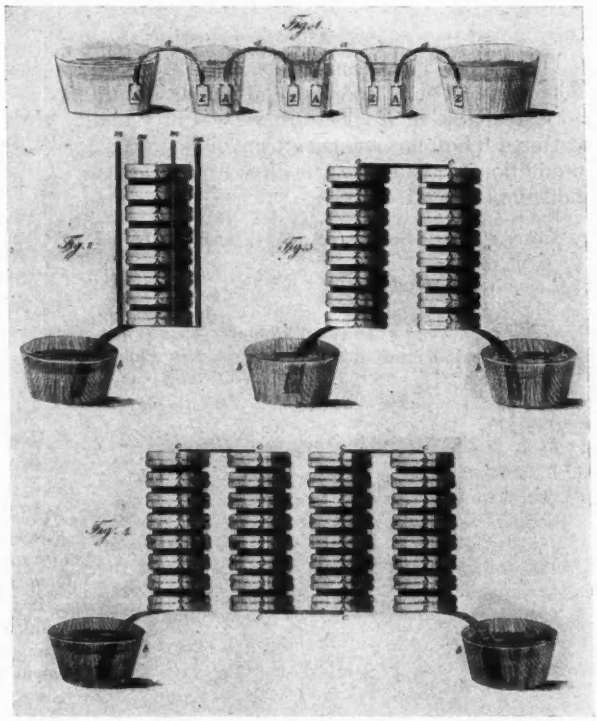

PLATE 1. Plate from Volta's famous paper "On the electricity excited by the mere contact of conducting substances of different kinds," Phil. Trans. 90, 403 (1800). [Courtesy of E. R. Hoge, Mount Wilson Observatory.]

[2] The original letter has been reproduced in facsimile by Sarton in *Isis*, reference 1. English translations of portions of it will be found in *A source book in physics*, by W. F. Magie (McGraw-Hill, 1935), pp. 427–431; in *A history of science, technology, and philosophy in the 18th Century*, by A. Wolf (Macmillan, 1939), pp. 260–265; in *The rise of the electrical industry during the nineteenth century*, by Malcolm MacLaren (Princeton, 1943); and elsewhere.

PLATE 2. Photograph of one of Volta's original piles.

There does not seem to be any standard biography of VOLTA, and the details of his life and work available in English are very scanty and scattered. The following incomplete bibliography, in addition to the references already cited, may therefore be of interest:

F. Walker, "Volta," Engineering **145**, 1–2, 53–54, 113–114 (1938).

E. C. Smith, "Alessandro Volta, 1745–1827," Nature **155**, 473 (1945).

D. F. J. Arago, *Éloge historique d'Alexandre Volta* (Paris, 1833). English translation in Edinburgh New Phil. J. **16**, 1–33 (1834).

P. Lenard, *Great men of science* (New York, 1933), pp. 158–170.

J. Munro, *Pioneers of electricity* (London, 1890).

M. F. O'Reilly and J. J. Walsh, *Makers of electricity* (Fordham Univ. Press, 1909).

P. Riccardi, *Sulle opere di Alessandro Volta, note bibliografiche* (Modena, 1877). List of 90 papers referring to the life and work of Volta.

F. Massardi, *L'Opera di Alessandro Volta* (Milan, 1927). Contains biographical notes as well as a selection from Volta's works.

G. Cau, *Alessandro Volta—l'uomo—la sua scienza—il suo tempo* (Milan, 1927).

E. Fabietti, *Alessandro Volta* (Milan, 1927).

G. Furlani, "L'Opera di Alessandro Volta nell' invenzione e nello studio della pila," Periodico Matematiche **7**, 154–186 (1927).

A. Mieli, *Alessandro Volta* (Rome, 1927).

G. Polvani, *Alessandro Volta* (Pisa, 1942).

C. Volpati, *Alessandro Volta nella gloria e nell' intimita* (Milan, 1927).

L. Volta, "La figura di Alessandro Volta, un secolo dopo," Boll. Matematica **24**, 1–19 (1928).

Della vita del Conte Alessandro Volta patrizio comasco (Como, 1829).

P. Janet, *Discours prononcé au Congrès international des physiciens tenu en commémoration du centenaire de la mort de Alessandro Volta* (Paris, 1927).

Nel 1° centenario della morte di Alessandro Volta, 1827–1927. Fasciolo speciale de l'energia elettrica (Milan, 1927).

A collection of VOLTA's works was published at Florence in 1816 and a sumptuous national edition at Milan beginning in 1918.

VOLTA'S LETTER

I. On the Electricity excited by the mere Contact of conducting Substances of different Kinds. In a Letter from Mr. ALEXANDER VOLTA, F.R.S. Professor of Natural Philosophy in the University of Pavia, to the Right Hon. Sir JOSEPH BANKS, Bart. K.B. P.R.S.*

Como in the Milanese, March 20, 1800.

AFTER a long silence, for which I shall offer no apology, I have the pleasure of communicating to you, and through you to the Royal Society, some striking results I have obtained in pursuing my experiments on electricity excited by the mere mutual contact of different kinds of metal, and even by that of other conductors, also different from each other, either liquid or containing some liquid, to which they are properly indebted for their conducting power. The principal of these results, which comprehends nearly all the rest, is the construction of an apparatus having a resemblance in its effects (that is to say, in the shock it is capable of making the arms, &c. experience) to the Leyden flask, or, rather, to an electric battery weakly charged acting incessantly, which should charge itself after each explosion; and, in a word, which should have an inexhaustible charge, a perpetual action or impulse on the electric fluid; but which differs from it essentially both by this continual action, which is peculiar to it, and because, instead of consisting, like the common electric

* Translated from the author's paper published in French in the Philosophical Transactions for 1800, part 2.

jars and batteries, of one or more insulating plates or thin strata of those bodies which are alone thought to be *electric*, armed with conductors, or bodies called *non-electric*, this new apparatus is formed merely of several of the latter bodies, chosen from among those which are the best conductors, and therefore the most remote, as has hitherto been believed, from the electric nature. The apparatus to which I allude, and which will, no doubt, astonish you, is only the assemblage of a number of good conductors of different kinds arranged in a certain manner. Thirty, forty, sixty, or more pieces of copper, or rather silver, applied each to a piece of tin, or zinc, which is much better, and as many strata of water, or any other liquid which may be a better conductor, such as salt water, ley, &c. or pieces of pasteboard, skin, &c. well soaked in these liquids; such strata interposed between every pair of combination of two different metals in an alternate series, and always in the same order of these three kinds of conductors, are all that is necessary for constituting my new instrument, which, as I have said, imitates the effects of the Leyden flask, or of electric batteries, by communicating the same shock as these do; but which, indeed, is far inferior to the activity of these batteries when highly charged, either in regard to the force and noise of the explosions, the spark, the distance at which the discharge may be effected, &c. as it equals only the effects of a battery very weakly charged, though of immense capacity: in other respects, however, it far surpasses the virtue and power of these batteries, as it has no need, like these, of being previously charged by means of foreign electricity, and as it is capable of giving a shock every time it is properly touched, however often it may be.

To this apparatus, much more similar at bottom, as I shall show, and even such as I have constructed it, in its form to the *natural electric organ* of the torpedo or electric eel, &c. than to the Leyden flask and electric batteries, I would wish to give the name of the *artificial electric organ:* and, indeed, is it not, like it, composed entirely of conducting bodies. Is it not also active of itself without any previous charge, without the aid of any electricity excited by any of the means hitherto known? Does it not act incessantly, and without intermission? And, in the last place, is it not capable of giving every moment shocks of greater or less strength, according to circumstances—shocks which are renewed by each new touch, and which, when thus repeated or continued for a certain time, produce the same torpor in the limbs as is occasioned by the torpedo, &c.?

I shall now give a more particular description of this apparatus and of others analogous to it, as well as of the most remarkable experiments made with them.

I provide a few dozens of small round plates or disks of copper, brass, or rather silver, an inch in diameter more or less (pieces of coin for example), and an equal number of plates of tin, or, what is better, of zinc, nearly of the same size and figure. I make use of the term *nearly*, because great precision is not necessary, and the size in general, as well as the figure of the metallic pieces, is merely arbitrary: care only must be taken that they may be capable of being conveniently arranged one above the other, in the form of a column. I prepare also a pretty large number of circular pieces of pasteboard, or any other spongy matter capable of imbibing and retaining a great deal of water or moisture, with which they must be well impregnated in order to ensure success to the experiments. These circular pieces of pasteboard, which I shall call moistened disks, I make a little smaller than the plates of metal, in order that, when interposed between them, as I shall hereafter describe, they may not project beyond them.

Having all these pieces ready in a good state, that is to say, the metallic disks very clean and dry, and the non-metallic ones well-moistened with common water, or, what is much better, salt water, and slightly wiped that the moisture may not drop off, I have nothing to do but to arrange them, a matter exceedingly simple and easy.

I place then horizontally, on a table or any other stand, one of the metallic pieces, for example one of silver, and over the first I adapt one of zinc; on the second I place one of the moistened disks, then another plate of silver followed immediately by another of zinc, over which I place another of the moistened disks. In this manner I continue coupling a plate of silver with one of zinc, and always in the same order, that is to say, the silver below and the zinc above it, or *vice versa*, according as I have begun, and interpose between each of these couples a moistened disk. I continue to form, of several of these stories, a column as high as possible without any danger of its falling.

But, if it contain about twenty of these stories or couples of metal, it will be capable not only of emitting signs of electricity by Cavallo's electrometer, assisted by a condenser, beyond ten or fifteen degrees, and of charging this

PLATE 3. Alessandro Volta.

condenser by mere contact so as to make it emit a spark, &c. but of giving to the fingers with which its extremities (the bottom and top of the column) have been touched several small shocks, more or less frequent, according as the touching has been repeated. Each of these shocks has a perfect resemblance to that slight shock experienced from a Leyden flask weakly charged, or a battery still more weakly charged, or a torpedo in an exceedingly languishing state, which imitates still better the effects of my apparatus by the series of repeated shocks which it can continually communicate.

To obtain such slight shocks from this apparatus which I have described, and which is still too small for great effects, it is necessary that the fingers, with which the two extremities are to be touched at the same time, should be dipped in water, so that the skin, which otherwise is not a good conductor, may be well moistened. To succeed with more certainty, and receive stronger shocks, a communication must be made, by means of a metallic plate sufficiently large, or a large metallic wire, between the bottom of the column (that is to say, the lower piece of metal), and water contained in a bason or large cup, in which one, two, or three fingers, or the whole hand is to be immersed, while you touch the top or upper extremity (the uppermost or one of the uppermost plates of the column) with the clean extremity of another metallic plate held in the other hand, which must be very moist, and embrace a large surface of the plate held very fast. By proceeding in this manner, I can obtain a small pricking or slight shock in one or two articulations of a finger immersed in the water of the bason, by touching, with the plate grasped in the other hand, the fourth or even third pair of metallic pieces. By touching then the fifth, the sixth, and the rest in succession till I come to the last, which forms the head of the column, it is curious to observe how the shocks gradually increase in force. But this force is such, that I receive from a column formed of twenty pairs of pieces (not more) shocks which affect the whole finger with considerable pain if it be immersed alone in the water of the bason; which extend (without pain) as far as the wrist, and even to the elbow, if the whole hand, or the greater part of it, be immersed; and are felt also in the wrist of the other hand.

I still suppose that all the necessary attention has been employed in the construction of the column, and that each pair or couple of metallic pieces, resulting from a plate of silver applied over one of zinc, is in communication with the following couple by a sufficient stratum of moisture, consisting of salt water rather than common water, or by a piece of pasteboard, skin, or any thing of the same kind well impregnated with this salt water. The disk must not be too small, and its surface must adhere closely to those of the metallic plates between which it is placed. This exact and extensive application of moistened disks is very important, whereas the metallic plates of each pair may only touch each other in a few points, provided that their contact is immediate.

All this shows that, if the contact of the metals with each other in some points only be sufficient (as they are excellent conductors) to give a free passage to a moderately strong current of electricity, the case is not the same with liquids, or bodies impregnated with moisture, which are conductors much less perfect; and which, consequently, have need of more ample contact with metallic conductors, and still more with each other, in order that the electric fluid may easily pass, and that it may not be too much retarded in its course; especially when it is moved with very little force, as in the present case.

In a word, the effects of my apparatus, that is to say, the shocks felt, are considerably more sensible in proportion as the temperature of the ambient air, or that of the water or moistened disks which enter into the composition of the column, and that of the water even in the bason, is warmer, as heat renders the water a better conductor. But almost all the salts, and particularly common salt, will render it a still better. This is one of the reasons, if not the only one, why it is so advantageous that the water of the bason, and, above all, that interposed between each pair of metallic plates, as well as the water with which the circular pieces of pasteboard are impregnated, &c. should be salt water, as already observed.

But all these means and all these attentions have only a limited advantage, and will never occasion your receiving very strong shocks as long as the apparatus consists but of one column, formed only of twenty pair of plates, even though they may consist of the two metals properest for these experiments, *viz.* silver and zinc; for if they were silver and lead, or tin, or copper and tin, the half of the effect would not be produced, unless the weaker effect of each pair were supplied by a much greater number. What really increases the electric power of this apparatus, and to such a degree as to make it equal or surpass that of the torpedo or electric eel, is the number of plates arranged in such a manner, and with the attention before mentioned. If to the twenty pairs above described twenty or thirty others be added disposed in the same order, the shocks which may be communicated by a column lengthened in this manner will be much stronger, and extend to both arms as far as the shoulder; and especially of that, the hand of which has been immersed in the water: this hand, with the whole arm, will remain more or less benumbed, if by frequently renewing the touches these shocks be made to succeed each other rapidly, and without intermission. This will be the case if the whole hand, or the greater part of it, be immersed in the water of the bason; but if only one finger be immersed, either wholly or in part, the shocks being almost entirely concentrated in it alone, will become so much the more painful, and so acute as to be scarcely supportable.

It may readily be conceived that this column, formed of forty or fifty couples of metals, which gives shocks more than moderate to both the arms of one person, is capable of giving sensible shocks also to several persons, holding each other by the hands (sufficiently moist) so as to form an uninterrupted chain.

I shall now return to the mechanical construction of my apparatus, which is susceptible of several variations, and describe not all those which I have invented or made, either on a small or a large scale, but only a few, which are either curious or useful, which exhibit some real ad-

vantage, as being easier or sooner constructed, and which are certain in their effects, or can be longer preserved in good order.

I shall begin by one which, uniting nearly all these advantages, differs most in its figure from the columnar apparatus above described, but which is attended with the inconvenience of being much more voluminous. This new apparatus, which I shall call a *couronne de tasses* (a chain of cups), is represented Plate VIII. fig. 1.*

I dispose, therefore, a row of several basons or cups of any matter whatever, except metal, such as wood, shell, earth, or rather glass (small tumblers or drinking glasses are the most convenient), half filled with pure water, or rather salt water or ley: they are made all to communicate by forming them into a sort of chain, by means of so many metallic arcs, one arm of which, *Sa*, or only the extremity *S*, immersed in one of the tumbers, is of copper or brass, or rather of copper plated with silver; and the other, *Za*, immersed into the next tumber, is of tin, or rather of zinc. I shall here observe, that ley and other alkaline liquors are preferable when one of the metals to be immersed is tin: salt water is preferable when it is zinc. The two metals of which each arc is composed, are soldered together in any part above that which is immersed in the liquor, and which must touch it with a surface sufficiently large: it is necessary therefore that this part should be a plate of an inch square, or very little less; the rest of the arc may be as much narrower as you choose, and even a simple metallic wire. It may also consist of a third metal different from the two immersed into the tumblers, since the action on the electric fluid which results from all the contacts of several metals that immediately succeed each other, or the force with which this fluid is at last impelled, is absolutely the same, or nearly so, as that which it would have received by the immediate contact of the first metal with the last without any intermediate metals, as I have ascertained by direct experiments, of which I shall have occasion to speak hereafter.

A series of 30, 40, or 60 of these tumblers connected with each other in this manner, and ranged either in a straight or curved line, or bent in every manner possible, forms the whole of this new apparatus, which at bottom and in substance is the same as the other columnar one above described; as the essential part, which consists in the immediate communication of the different metals which form each couple, and the immediate communication of one couple with the other, *viz.* by the intervention of a humid conductor, exist in the one as well as the other.

In regard to the manner of trying these tumblers, and the different experiments for which they may be employed, there is no need of saying a great deal after the ample explanation I have already given respecting the columnar apparatus. It may be readily comprehended, that to obtain a shock it will be sufficient to immerse one hand into one of the tumblers, and a finger of the other hand into another of the tumblers at a considerable distance from the former: that this shock will be stronger the further these glasses are from each other; that is to say, in proportion to the number of the intermediate glasses, and consequently, that the strongest shock will be received when you touch the first and last end of the chain. It will be readily comprehended also, how and why the experiments will succeed much better by grasping and holding fast in one hand, well moistened, a pretty large plate of metal (in order that the communication may be more perfect, and formed in a great number of points), and touching with this plate the water in the tumbler, or rather the metallic arc, while the other is immersed in the other distant tumbler, or touches with a plate, grasped in the like manner, the arc of the latter. In a word, one may comprehend and even foresee the success of a great variety of experiments which may be made with this apparatus or chain of cups much more easily, and in a manner more evident, and which, if I may be allowed the expression, speak more to the eyes than those with the columnar apparatus. I shall therefore forbear from describing a great number of these experiments, which may be easily guessed, and shall relate only a few which are no less instructive than amusing.

Let three twenties of these tumblers be ranged, and connected with each other by metallic arcs, but in such a manner, that, for the first twenty, these arcs shall be turned in the same direction; for example, the arm of silver turned to the left, and the arm of zinc to the right; and for the second twenty in a contrary direction, that is to say, the zinc to the left, and the silver to the right: in the last place, for the third twenty, the silver to the left, as is the case in regard to the first. When every thing is thus arranged, immerse one finger in the water of the first tumbler, and, with the plate grasped in the other hand, as above directed, touch the first metallic arc (that which joins the first tumbler to the second), then the other arc which joins the second and third tumbler, and so on, in succession, till you have touched them all. If the water be very salt and luke-warm, and the skin of the hands well moistened and softened, you will already begin to feel a slight shock in the finger when you have touched the fourth or fifth arc (I have experienced it sometimes very distinctly by touching the third), and by successively proceeding to the sixth and the seventh, &c. the shocks will gradually increase in force to the twentieth arc, that is to say, to the last of those turned in the same direction; but by proceeding onwards to the 21 st, 22d, 23d, or 1 st, 2d, 3d, of the second twenty, in which they are all turned in a contrary direction, the shocks will each time become weaker, so that at the 36th or 37th, they will be imperceptible, and be entirely null at the 40th, beyond which (and beginning the third twenty, opposed to the second and analogous to the first,) the shocks will be imperceptible to the 44th or 45th arc; but they will begin to become sensible, and to increase gradually, in proportion as you advance to the 60th, where they will have attained the same force as that of the 20th arc.

If the twenty arcs in the middle were all turned in the same direction as the preceding twenty and the following twenty, that is to say, if the whole 60 conspired to impel the electric fluid in the same direction, it may readily be comprehended how much greater the effect will be at the end,

* See our Plate 1.—E. C. W.

and how much stronger the shock; and it may be comprehended, in general, to what point it must be weakened in all cases where a greater or smaller number of these forces act contrary to each other by an inverted position of metals.

If the chain be in any part interrupted, either by one of the tumblers being empty of water, or one of the metallic arcs being removed or divided into two pieces, you will receive no shock when you immerse your finger into the water of the first and another into that of the last vessel; but you will have it strong or weak, according to circumstances (leaving these fingers immersed), at the moment when the interrupted communication is restored; at the moment when another person shall immerse into the two tumblers, where the arc is wanting, two of his fingers (which will also receive a slight shock), or rather, when he shall immerse the same arc which has been taken away, or any other; and in the case of the arc separated into two pieces, at the moment when these pieces are again brought into mutual contact (in which case the shock will be stronger than in any other); and, lastly, in the case of the empty tumbler, at the moment when water poured into it shall rise to the two metallic arms immersed in this cup which before were dry.

When the chain of cups is of sufficient length, and capable of giving a strong shock, you will experience one, though much weaker, even though you keep immersed two fingers, or the two hands, in one bason of water of pretty large size, in which the first and last metallic arcs are made to terminate: provided that either of these hands thus immersed, or rather both of them, be kept respectively in contact, or nearly in contact, with these arcs, you will, I say, experience a shock at the moment when (the chain being interrupted in any part) the communication is restored, and the circle completed in any of the ways before mentioned. One might be surprised that in this circle the electric current having a free passage through an uninterrupted mass of water, that which fills the bason, should quit this good conductor to throw itself and pursue its course through the body of the person who holds his hands immersed in the same water, and thus to take a longer passage. But the surprise will cease if we reflect that living and warm animal substances, and above all, their humours, are, in general, better conductors than water. As the body, then, of the person who immerses his hands in the water, affords an easier passage than this water does to the electric current, the latter must prefer it though a little longer. In a word, the electric fluid, when it must traverse imperfect conductors in a large quantity, and particularly moist conductors, has a propensity to extend itself in a larger stream, or to divide itself into several, and even to pursue a winding course, as it thereby finds less resistance than by following one single channel, though shorter; in the present case it is only a part of the electric current, which, leaving the water, pursues this new route through the body of the person, and traverses it from the one arm to the other: a greater or less part passes through the water in the vessel. This is the reason why the shock experienced is much weaker than when the electric current is not divided when the person alone forms the communication between one arc and another, &c.

From these experiments one might believe, that when the torpedo wishes to communicate a shock to the arms of a man or to animals which touch it, or which approach its body under the water (which shock is much weaker than what the fish can give out of the water), it has nothing to do but to bring together some of the parts of its electric organ in that place, where, by some interval, the communication is interrupted, to remove the interruptions from between the columns of which the said organ is formed, or from between its membranes in the form of thin disks, which lie one above the other from the bottom to the summit of each column: it has, I say, nothing to do but to remove these interruptions in one or more places, and to produce there the requisite contact, either by compressing these columns, or by making some moisture to flow in between the pellicles or diaphragms which have been separated, &c. This is what may be, and what I really conclude to be, the task of the torpedo when it gives a shock; for all the rest, the impulse and movement communicated to the electric fluid, is only a necessary effect of its singular organ, formed, as is seen, of a very numerous series of conductors, which I have every reason to believe sufficiently different from each other to be *exciters* of the electric fluid by their mutual contacts; and to suppose them ranged in a manner proper for impelling that fluid with a sufficient force from top to bottom, or from the bottom to the top, and for determining a current capable of producing the shock, &c. as soon and as often as all the necessary contacts and communications take place.

But let us now leave the torpedo, and its *natural electric organ*, and return to the *artificial electric organ* of my invention, and particularly to my first *columnar apparatus*, that which imitates the first even in its form (for that composed of tumblers is different in that respect). I might say something also in regard to the construction of the said apparatus with tumblers or *a chain of glasses;* for example, that the first and last tumbler should be of such a size that, when necessary, the whole hand might be immersed in it, &c.; but, to enter into all these details, would require too much time.

In regard to the columnar apparatus, I endeavoured to discover the means of lengthening it a great deal by multiplying the metallic plates in such a manner as not to tumble down; and I discovered, besides others, the following, which are represented in the annexed figures. (Plate VIII. Fig. 2, 3, 4.)

In Fig. 2, *mmmm* are rods, three, four, or more in number, which rise from the bottom of the column, and confine, as in a cage, the plates or disks, placed each above the other in such number and to such a height as you choose, and which thus prevent them from falling. The rods may be of glass, wood, or metal, only that, in the last case, you must prevent them from coming into immediate contact with the plates; which may be done either by covering each of them with a glass tube, or interposing between them and the column a few stripes of wax cloth, oiled paper, or even plain paper, and, in a word, any other body that may either be a *cohibent* or a bad conductor:

wood or paper will be sufficiently so for our purpose, provided only that they are not very damp or moist.

But the best expedient, when you wish to form an apparatus to consist of a great number of plates, above 60, 80, or 100 for example, is, to divide the column into two or more, as seen Fig. 3 and 4, (Plate VIII.) where the pieces all have their respective positions and communication as if there were only one column. Fig. 4, as well as Fig. 3, may indeed be considered as a bent column.

In all these figures the different metallic plates are denoted by the letters *S* and *Z* (which are the initials of silver and zinc); and the *moistened disks* (of pasteboard, skin, &c. interposed between each pair of metals), are represented by a black stratum. The plates of metal may either be laid simply upon each other and so brought into union in an indefinite number of points, or they may be soldered together. It is altogether indifferent whichever of these methods be followed. *cc*, *cc*, *cc*, are the metallic plates which form a communication between each column, or section of a column, and another; and *bb*, *bb*, *bb*, are the basons of water in communication with the lower part or extremities of these columns.

An apparatus thus prepared is exceedingly convenient without being bulky; and it might be rendered portable, with still more ease and safety, by means of circular cases or tubes, in which each column might be inclosed and preserved. It is only to be regretted that it does not long continue in a good state: the moistened disks become dry in one or two days to such a degree that they must be again moistened; which, however, may be done without taking to pieces the whole apparatus, by immersing the columns, completely formed, in water, and wiping them, when taken out some time after, with a cloth, or in any other manner.

The best method of making an instrument as durable as can be wished, would be, to inclose and confine the water interposed between each pair of metals, and to fix these metallic plates in their places by enveloping the whole column with wax or pitch: but this would be somewhat difficult in the execution, and would require a great deal of patience. I have, however, succeeded; and have formed in this manner two cylinders consisting of twenty pair of metals, which can still be employed though made several weeks, and which, I hope, will be serviceable for months.

These cylinders are attended with this advantage, that they may be employed for experiments either in an erect, inclined, or lying position, according as you choose, or even immersed in water, provided the top of it be above the surface of the fluid: they might also give a shock when entirely immersed if they contained a greater number of plates, or if several of these cylinders were joined together, and if there were any interruption that could be removed at pleasure, &c. by which means these cylinders would have a pretty good resemblance to the electric eel; and, to have a better resemblance to it even externally, they might be joined together by pliable metallic wires or screw-springs, and then covered with a skin terminated by a head and tail properly formed, &c.

The effects sensible to our organs produced by an apparatus formed of 40 or 50 pair of plates (and even by a smaller, if one of the metals be silver or copper and the other zinc,) are reduced merely to shocks: the current of the electric fluid, impelled and excited by such a number and variety of different conductors, silver, zinc, and water, disposed alternately in the manner above described, excites not only contractions and spasms in the muscles, convulsions more or less violent in the limbs through which it passes in its course; but it irritates also the organs of taste, sight, hearing, and feeling, properly so called, and produces in them sensations peculiar to each.

And first, in regard to the sense of feeling: If, by means of an ample contact of the hand (well moistened) with a plate of metal, or rather, by immersing the hand to a considerable depth in the water of the bason, I establish on one side a good communication with one of the extremities of my *electro-motive* apparatus, (we must give new names to instruments that are new not only in their form, but in their effects or the principle on which they depend); and on the other I apply the forehead, eye-lid, tip of the nose, also well-moistened, or any other part of the body where the skin is very delicate: if I apply, I say, with a little pressure, any one of these delicate parts, well-moistened, to the point of a metallic wire, communicating properly with the other extremity of the said apparatus, I experience, at the moment that the conducting circle is completed, at the place of the skin touched, and a little beyond it, a blow and a prick, which suddenly passes, and is repeated as many times as the circle is interrupted and restored; so that, if these alternations be frequent, they occasion a very disagreeable quivering and pricking. But if all these communications continue without these alternations, without the least interruption of the circle, I feel nothing for some moments; afterwards, however, there begins at the part applied to the end of the wire, another sensation, which is a sharp pain (without shock), limited precisely by the points of contact, a quivering, not only continued, but which always goes on increasing to such a degree, that in a little time it becomes insupportable, and does not cease till the circle is interrupted.

What proof more evident of the continuation of the electric current as long as the communication of the conductors forming the circle is continued?—and that such a current is only suspended by interrupting that communication? This endless circulation of the electric fluid (this *perpetual motion*) may appear paradoxical and even inexplicable, but it is no less true and real; and you feel it, as I may say, with your hands. Another evident proof may be drawn from this circumstance, that in such experiments you often experience, at the moment when the circle is suddenly interrupted, a shock, a pricking, an agitation, according to circumstances, in the same manner as at the moment when it is completed; with this only difference, that these sensations, occasioned by a kind of reflux of the electric fluid, or by the shock which arises from the sudden suspension of its current, are of less strength. But

* Our Plate 1 is reproduced from the original paper in the *Philosophical Transactions*, which is in French and therefore uses A for silver.—E. C. W.

I have no need, and this is not the place to bring forward proofs of such an endless circulation of the electric fluid in a circle of conductors, where there are some, which, by being of a different kind, perform, by their mutual contact, the office of exciters or *movers:* this proposition, which I advanced in my first researches and discoveries on the subject of galvanism, and always maintained by supporting them with new facts and experiments, will, I hope, meet with no opposers.

Recurring to the sensation of pain which is felt in the experiments above described, I must add, that if this pain be very strong and pricking in the parts covered by the skin, it is much more so in those where the skin has been taken off—in recent wounds for example. If by chance there should be a small incision or bit of the skin rubbed off in the finger which I immerse in the water that communicates with one of the extremities of the *electro-motive* apparatus, I experience there a pain so acute, when, by establishing the proper communication with the other extremity, I complete the circle, that I must soon desist from the experiment; that is to say, must withdraw my finger, or interrupt the circle in some other manner. I will say more; that I cannot even endure it above a few seconds when the part of the apparatus which I put in play, or the whole apparatus, contains only twenty pair of plates, or about that number.

One thing, which I must still remark, is, that all these sensations of pricking and pain are stronger and sharper, every thing else being equal, when the part of the body which is to feel them is towards the negative electricity; that is to say, placed in such a manner in the conducting circle, that the electric fluid traversing that circle is not directed towards that sensible part, does not advance towards it, and enter from the outside inwards, but takes its direction from the inside outwards; in a word, that it issues from it: in regard to which it is necessary to know, of the two metals that enter by pairs into the construction of the machine, which is the one that gives off to the other. But I had already determined this respecting all the metals by other experiments, published a long time ago at the end of my first memoirs on galvanism. I shall therefore say nothing further here, than that the whole is completely confirmed by the experiments, equally and still more demonstrative and striking, with which I am at present employed.

In regard to the sense of taste, I had before discovered, and published in these first memoirs, where I found myself obliged to combat the pretended animal electricity of Galvani, and to declare it an external electricity moved by the mutual contact of metals of different kinds,—I had discovered, I say, in consequence of this power which I ascribed to metals, that two pieces of these different metals, and particularly one of silver and one of zinc, applied in a proper manner, excited at the tip of the tongue very sensible sensations of taste; that the taste was decidedly acid, if, the tip of the tongue being turned towards the zinc, the electric current proceeded against it, and entered it; and that another taste, less strong but more disagreeable, acrid, and inclining to alkaline, was felt, if (the position of the metals being reversed) the electric current issued from the tip of the tongue; that these sensations continued and received even an increase for several seconds, if the mutual contact of the two metals was maintained, and if the conducting circle was nowhere interrupted. But when I have said here, that exactly the same phenomena take place when you try, instead of one pair of these metallic pieces, an assemblage of several of them ranged in the proper manner; and that the said sensations of taste, whether acid or alkaline, increase but a little with the number of these pairs, I have said the whole. It only remains for me to add that, if the apparatus put in play for these experiments on the tongue be formed of a sufficiently large number of metallic pairs of this kind, for example, if it contain 30, 40, or more, the tongue experiences not only the sensation of taste already mentioned, but, besides that, a blow which it receives at the moment when the circle is completed, and which occasions in it a pricking more or less painful, but fleeting, followed some moments after by a durable sensation of taste. This blow produces even a convulsion or agitation of a part or of the whole of the tongue, when the apparatus, formed of a still greater number of pairs of the said metals, is more active, and if, by means of good communicating conductors, the electric current which it excites be able to pass every where with perfect freedom.

I must often recur to, and insist on, this last condition, because it is essential in all experiments when you wish to obtain sensible effects on the body, or commotions in the limbs, or sensations in the organs of the senses. It is necessary, therefore, that the non-metallic conductors which enter into the circle should be as good conductors as possible, well-moistened (if they are not themselves liquid) with water, or with any other liquid that may be a better conductor than pure water; and it is necessary, besides, that the well-moistened surfaces, by which they communicate with the metallic conductor, should be sufficiently large. The communication ought to be confined or reduced to a small number of points of contact only in that place where you wish to concentrate the electric action on one of the most sensible parts of the body, on any of the sensitive nerves, &c. as I have already remarked in speaking of the experiments on feeling, *viz.* those by which acute pains are excited in different parts. The best method which I have found for producing on the tongue all the sensations above described, is, to apply the tip of it to the pointed extremity (which, however, must not be too much so) of a metallic rod, which I make to communicate properly, as in the other experiments, with one of the extremities of my apparatus, and to establish a good communication between the hand, or, what is better, both the hands together, and the other extremity. This application of the tip of the tongue to the end of the metallic rod, may either exist already, when you are going to make the other communication to complete the circle (when you are going to immerse your hand into the water of the bason), or be made after the establishment of this communication, while the hand is immersed; and in the latter case I think I feel the pricking and shock in the tongue, a very short time

before actual contact. Yes; it always appears to me, particularly if I advance the tip of my tongue gradually, that, when it has arrived within a very small distance of the metal, the electric fluid (I would almost say spark), overcoming this interval, darts forwards to strike it.

In regard to the sense of sight, which I also found might be affected by the weak current of the electric fluid, arising from the mutual contact of two different metals in general, and in particular, of a piece of silver and one of zinc, it was natural to expect that the sensation of light, excited by my new apparatus, would be stronger in proportion as it contained a greater number of pieces of these metals; each pair of which, arranged in the proper manner, adds a degree of force to the said electric current, as all the other experiments show, and particularly those with the electrometer assisted by the condenser, which I have only mentioned, and which I shall describe on another occasion. But I was surprised to find that, with 10, 20, 30 pairs, and more, the flash produced neither appeared longer and more extended, nor much brighter than with one pair. It is true, however, that this sensation of weak and transient light, is excited by such an apparatus much easier and in different ways. To succeed, indeed, with one pair, the following are almost the only methods; *viz.* that one of the metallic pieces should be applied to the ball of the eye, or the eye-lid well-moistened, and that it should be made to touch the other metal applied to the other eye, or held in the mouth, which produces a flash much more beautiful; or, that this second metallic piece should be held in the moistened hand and then brought into contact with the former; or, in the last place, that these two plates should be applied to certain parts of the inside of the mouth, making them communicate with each other. But with an apparatus of 20 or 30 pairs, &c. the same flash will be produced by applying the end of a metallic plate or rod, placed in communication with one of the extremities of the apparatus, to the eye, while with one hand you form a proper communication with the other extremity; by bringing, I say, this plate into contact not only with the eye or any part of the mouth, but even the forehead, the nose, the cheeks, lips, chin, and even the throat; in a word, every part and point of the visage, which must only be well moistened before they are applied to the metallic plate. The form as well as the force of this transient light which is perceived varies a little, if the places of the face to which the action of the electric current is applied, be varied: if it be on the forehead, for example, this light is moderately bright, and appears like a luminous circle, under which figure it presents itself also in several other experiments.

But the most curious of all these experiments is, to hold the metallic plate between the lips, and in contact with the tip of the tongue; since, when you afterwards complete the circle in the proper manner, you excite at once, if the apparatus be sufficiently large and in good order, and the electric current sufficiently strong and in good order, a sensation of light in the eyes, a convulsion in the lips, and even in the tongue, and a painful prick at the tip of it, followed by a sensation of taste.

I have now only to say a few words on hearing. This sense, which I had in vain tried to excite with only two metallic plates, though the most active of all the *exciters* of electricity, *viz.* one of silver or gold, and the other of zinc, I was at length able to affect it with my new apparatus, composed of 30 or 40 pairs of these metals. I introduced, a considerable way into both ears, two probes or metallic rods with their ends rounded, and I made them to communicate immediately with both extremities of the apparatus. At the moment when the circle was thus completed I received a shock in the head, and some moments after (the communication continuing without any interruption) I began to hear a sound, or rather noise, in the ears, which I cannot well define: it was a kind of crackling with shocks, as if some paste or tenacious matter had been boiling. This noise continued incessantly, and without increasing, all the time that the circle was complete, &c. The disagreeable sensation, and which I apprehended might be dangerous, of the shock in the brain, prevented me from repeating this experiment.

There still remains the sense of smelling, which I have hitherto tried in vain with my apparatus. The electric fluid, which, when made to flow in a current in a complete circle of conductors, produces in the limbs and parts of the living body effects correspondent to their excitability, which stimulating in particular the organs or nerves of touch, taste, sight, and hearing, excite in them some sensations peculiar to each of these senses, as I have found, produces in the interior of the nose only a pricking more or less painful, and commotions more or less extensive, according as the said current is weaker or stronger. And whence comes it, then, that it does not excite any sensation of smell, though, as appears, it stimulates the nerves of that sense? It cannot be said that the electric fluid of itself is not proper for producing odorous sensations, since, when it diffuses itself through the air in the form of aigrettes, &c. in the common experiments made with electric machines, it conveys to the nose a very sensible smell resembling that of phosphorus. Taking similitude into consideration, and reasoning from its analogy with other odoriferous matters, I will say, that it must completely diffuse itself throughout the air to excite smell; that it has need, like other effluvia, of the vehicle of the air to affect that sense in such a manner as to excite the sensations of smell. But in the experiments of which I speak, that is to say, of an electric current in a circle of conductors, all contiguous, and without the least interruption, this absolutely cannot take place.

All the facts which I have related in this long paper in regard to the action which the electric fluid excited, and when moved by my apparatus, exercises on the different parts of our body which the current attacks and passes through;—an action which is not momentaneous, but which lasts, and is maintained during the whole time that this current can follow the chain not interrupted in its communications; in a word, an action the effects of which vary according to the different degrees of excitability in the parts, as has been seen;—all these facts, sufficiently numerous, and others which may be still discovered by multiplying and varying the experiments of this kind, will open a very wide field for reflection, and of views, not only

curious, but particularly interesting to medicine. There will be a great deal to occupy the anatomist, the physiologist, and the practitioner.

It is well known, by the anatomy which has been made of it, that the electric organ of the torpedo or electric eel, consists of several membranaceous columns, filled from one end to the other with a great number of plates or pellicles, in the form of very thin disks, placed one upon the other, or supported at very small distances by intervals, into which, as appears, some liquor flows. But we cannot suppose that any of these laminæ are of an insulating nature, like glass, resin, silk, &c. and still less that they can either become electric by friction, or be disposed and charged in the same manner as the small Franklinian plates or small electrophores; nor even that they are sufficiently bad conductors to perform the office of a good and durable condenser, as Mr. Nicholson has supposed. The hypothesis of this learned and laborious philosopher, by which he makes of each pair of these pellicles, which he compares to leaves of talc, as many small *electrophores* or *condensers*, is indeed very ingenious, and is, perhaps, the best theory that has been devised to explain the phenomena of the torpedo, adhering to the hitherto known principles and laws of electricity. For, besides that the mechanism, by which, every time that the fish intended to give a shock, the respective separation of the plates of the whole or a great number of these electrophores or condensers ought to be effected all at once, and ought to establish on the one hand a communication between themselves of all the plates electrified *positively*, and on the other a communication between all those electrified *negatively*, as Mr. Nicholson supposes—besides, that this very complex mechanism appears too difficult, and little agreeable to nature;—and besides, that the supposition of an electric charge originally impressed, and so durable in these pellicles performing the office of electrophores, is altogether gratuitous,—such a hypothesis falls entirely, since these pellicles of the organ of the torpedo are not, and cannot be, in any manner insulating or susceptible of a real electric charge, and much less capable of retaining it. Every animal substance, as long as it is fresh, surrounded with juices, and more or less succulent of itself, is a very good conductor. I say more, instead of being as *cohibent* as resins or talc, to leaves of which Mr. Nicholson has compared the pellicles in question, there is not, as I have assured myself, any living or fresh animal substance which is not a better *deferent* than water, except only grease and some oily humours. But neither these humours nor grease, especially semi-fluid or entirely fluid, as it is found in living animals, can receive an electric charge in the manner of insulating plates, and retain it: besides, we do not find that the pellicles and humours of the organ of the torpedo are greasy or oily. This organ therefore, composed entirely of conducting substances, cannot be compared either to the electrophore or condenser, or to the Leyden flask, or any machine excitable by friction or by any other means capable of electrifying insulating bodies, which before my discoveries were always believed to be the only ones originally electric.

To what electricity then, or to what instrument ought the organ of the torpedo or electric eel, &c. to be compared? To that which I have constructed according to the new principle of electricity, discovered by me some years ago, and which my successive experiments, particularly those with which I am at present engaged, have so well confirmed, *viz.* that conductors are also, in certain cases, exciters of electricity in the case of the mutual contact of those of different kinds, &c. in that apparatus which I have named the *artificial electric organ*, and which being at bottom the same as the natural organ of the torpedo, resembles it also in its form, as I have advanced.

The Electric Circuit Equivalent of a Sentence

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"IN the beginning God created the heaven and the earth." Then in the King James version of the Bible comes a period. The next sentence starts with the word "and," despite the howls of any conventional teachers of freshman English—and why shouldn't it? for that word is nothing more than the plus sign with which we have all been familiar since we started to study algebra; it is a connective used to conjoin word with word or clause with clause. In my own purism, I demand only that this conjunction shall be used to connect things of similar type—like stray thoughts or dicta—or, in general, things congruent.

But—and don't wince if I start not only this sentence but also its paragraph with this conjunction, which after all means "not withstanding." Notwithstanding my previous digression, let us get along with the main thought. "And the earth was without form and void; and darkness was upon the face of the deep. And the Spirit of God moved upon the face of the waters. And God said, Let there be light: and there was light."

Notice that those ideas, probably the most widely known of all statements and the introduction to the most potent literature ever written, are all expressed in short sentences.

Whether you agree that this was the way it happened in 4004 B.C. or prefer to consider the description as scientifically inadequate and say that any resemblance to the physical facts is entirely coincidental, you will admit that the story is clearly told in short, concise sentences. A short sentence may be factually wrong, but there is little excuse for its being obscure; and it rarely is. It is the longer and more involved structure that offers the greatest latitude, for misinterpretation, not only by its reader but primarily by its writer.

Of course, you may say that the scientific ideas of today are much more complex than those of the days when Genesis was composed. True; but it is not necessarily true that their expression requires more complicated and involved forms. For the moment, assume that science is the Jonah of the Bible, and that the purpose of the Bible was instruction in religion and ethics and not science. Saint Paul, its most philosophic expositor, wrote sentences which for conciseness and clarity can put to shame most modern writers in philosophy and ethics just as can the terseness of Tacitus most historians.

One might enter as a rejoinder the fact that a succession of short sentences may lose effect by a monotony of style; but that is not necessarily inherent in the sentence structure. On the other hand, long sentences may be lucid, but the probability that they are is small. These are matters for further consideration.

At the moment I want to call attention to the analogy between a sentence and an electric circuit. The most complicated circuits of today are those of the mechanisms that automatically handle telephone calls in response to dialed signals. These act with almost human intelligence and with something more than human reliability. The circuit arrangement which operates as an electric brain to receive your dialed command and route your call through its own office, or nearby offices, to its particular destination is an amazing complex of coils, capacitors, relays and voltage sources. Its wiring diagram requires scores of blueprints—so many in fact that the subject index of the necessary sheets may form a good-sized pamphlet. Those not trained in circuit design can quickly be lost in its maze; but all may perhaps recognize that the entire circuit is an interrelated assembly of simpler circuits and that each of these in turn is built up of elementary circuits.

Each elementary circuit consists of a source of electromotive force, a switch and a "load" where the emf does its work when released by the switch. In such complicated circuits as we are considering, the load is usually a relay, an electromagnetic switch. Its armature bears one or more contact points which open or close other circuits and thus controls in them sources of emf. It is through the relay connection that one circuit affects another. But the details need not be pursued further.

Important to my thesis is the fact that the elementary circuit is the "equivalent circuit" of the simplest sentence. In one there are an energy source, a switch which acts and a load on which the energy is expended. In the other there are subject, predicate and object; and the subject acts on the object. In the basic sentence and in its circuit equivalent the relationship of the parts and the meaning are obvious. There should be no difficulty for circuit designer or for writer in devising a simple circuit.

Difficulty arises when one circuit is connected to another or one simple sentence to another. The circuit designer must surmount this difficulty, for he must organize intricate successions of operations. The writer can avoid the difficulty, if he wishes, by making his discourse a series of easily understood simple sentences. The object of one can be the subject of the next; the statement of one may qualify that of another. Also a qualifying adverbial phrase can always be introduced into an elementary sentence; by analogy it corresponds to the inevitable resistance which is found alike in energy source, switch, intervening wires and load.

In a high frequency circuit the qualifying element is not limited to resistance but includes distributed inductance and capacitance, and may include radiation resistance. However, just as it is unwise to attempt to design high frequency circuits until one is adept in low frequency design, so probably it is the course of wisdom to confine one's writing to a logical arrangement of simple sentence structures until the possibilities of that style have been exhausted. By many writers the limits of this style, which I am now recommending, have not even been approached.

Its advantage is not only the resultant lucidity but also, and even more important, the framework which it provides the author for the organization of his thoughts. Each sentence stands out clearly for his own criticism. By simple adverbial phrases, as has been said, he can qualify a circuit as he desires. He can inject the words "except as

noted later" or "with accompanying effects as listed below" and then go on with a new sentence starting "an exception is" or "one simultaneously occurring effect is." However he does it, the important thing is to have his discourse a string of crystals and not an amorphous lump of obscurity.

Take a lesson from the Euclidean theorem. Is there any more easily intelligible form? Given A; given B; given whatever you wish. To find something or to prove something. And then the proof: X equals Y; it does because so-and-so. And so on to the conclusion. The clarity of its style is not lost if transitive verbs replace the intransitive equality sign and action takes place. *The advantage of the theorem is the definiteness with which it hangs together; no statement goes untested and none hides in sentence obscurity.* If an exposition has the logical sequence and clarity of the ordinary theorem, one need not worry about its lack of literary form or its short sentences; it will carry its message.

At this point it is necessary to insert a disclaimer. I am not inveighing against long words but rather against long sentences—at least until one can write with competence. I have no objection to long words and do not demand short Anglo-Saxon words. There are no short circumlocutions to replace such words as electromagnetic, capacitor, automobile, electrometer or cyclotron. Use the shortest word that expresses completely and exactly the meaning, even if it is a Latin or a Greek compound with technical significance. The clarity of writing depends upon the lucidity with which appears the relationship between its words.

Ordinarily, I believe, short sentences will save words as well as increase clarity. That is particularly so if one will admit the paragraph to be the unit for comprehension and not demand that each sentence shall tell the whole truth, nothing but the truth, and state all the qualifications including the kitchen sink. The usual legal sentence, for example, which includes all the sinks, is too involved to be clarified merely by a little punctuation. Incidentally, as one who sometimes has to read income tax laws and the fine print of insurance policies, I wish that lawyers could write their statements in mathematical form, since to most of us that would yield shorter sentences and fewer confusing connectives of qualification. When a lawyer needs an involved sentence of 300 words to express an idea and cannot do it in 15 sentences averaging 20 words, I think he should return to school to study not law but the Bible and Euclid.

There is no reason why a scientist should write like a lawyer, or like a poet either, for that matter. His is a task of exposition where there is an inherent logic to his findings. He can avoid a mass of unnecessary words if he will think clearly before he starts and will describe his data and state his conclusions in a logical sequence of sentences each of which is analogous to the elementary electric circuit.

Now, I am going to risk offering a few instructions for inexperienced writers who would produce good scientific exposition. Take careful note, however, that effective exposition cannot be accomplished without thought, a considerable expenditure of time and strong intention. Much of the poor writing which disgraces term papers in colleges and even the pages of scientific periodicals, probably is less the result of inability to think clearly than of unwillingness to spend the necessary time and effort. The authors would resent an imputation of laziness or sloppiness; but there is evidence of both in authors' frequent attitudes that their writing is good enough, because "the professor will understand" or "my fellow experts will understand" and, of course, "I understand what I meant when I wrote." You must spend time on your exposition.

If you are willing to do so, write out what you want to say, using only short simple sentences. Then check your qualifying adverbs. Next rearrange your sentences in either a more logical order or, preferably, a more psychological order. I am using the latter term to denote the order in which your reader will more easily apprehend your main idea, whereas a logical order is that in which material can be cast only after it is learned. Remember that exposition is a pedagogic operation performed to educate the reader, and it should be arranged for that purpose.[1]

Now reread, checking your qualifying adverbs. You may find that one of your sentences can be joined with another. Insert the proper connective, or substitute the proper relative pronoun, and swing that sentence into the body of the other. But don't do it if it disrupts the rhythm of your discourse, because you have already attained *clarity, which is the object of exposition.*

[1] For an elaboration of these and related ideas, see J. Mills, "Technical exposition for the general reader," J. Eng. Ed. 23, 354 (1933). For a reprint of this article, address the Librarian, Bell Telephone Laboratories, 463 West Street, New York 14, New York.

Alice and the Sluggers

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ALICE was sleepy. She was tired from pushing a slide rule all day, making calculations on a strange thing called jet propulsion. Her profession—a new one for girls—was making engineering computations.

But Alice was sleepy. Her thoughts of slide rules and figures seemed to blur and become meaningless; and they escaped from their difficult task and fled, as they often did, across the Atlantic to her happy childhood home in England. She liked her name, Alice, for, as far back as she could remember, her favorite heroine had been that other Alice who had those marvelous "Adventures in Wonderland" and "Through the Looking Glass." She had liked to think of herself as Alice, Jr. Later, at Oxford, her studies of mathematics and physics gained extra zest from the thought that she was following directly in the footsteps of the master, Lewis Carroll, whose whimsical brain had given birth to her heroine. Then she had been projected across the ocean to apply her mathematics to the feverish war work of Britain's great American ally.

But now it was not to an England torn by bombs that her sleepy thoughts turned, but to a peaceful England where whimsies of white rabbits led little girls to confusing but delightful adventure. Suddenly her wandering mind was recalled to her room, which had become strange. Beside her sat a man whom she dimly recognized as her engineering supervisor, but whose huge hat and de-intellectualized features revealed him clearly as the Mad Hatter himself.

"Alice, Jr.," began the Mad Hatter, "your work is seriously defective. You should make that computation in slugs."

"But," said Alice, Jr., "a slug is a slimy thing like a snail, that eats cabbages."

"Well," said the Mad Hatter, "some people say this kind of a slug is slimy all right, but it isn't like a snail. It is what you get if you use pounds-force, feet and seconds, when you divide weight by gravitational acceleration. This is W divided by g and tells how much mass you have. I suppose we could define mass with any units of force and length, but most people use pounds-force, feet and seconds, and get slugs."

"We never learned that in England," said Alice. "We learned that mass was the muchness of anything, like how much sugar you get for sixpence and a ration stamp, and often we called it *weight* instead of mass. I think of muchness as meaning quantity of matter, which is the same, or nearly the same, as inertial mass. And when you say 'gravitational acceleration,' I suppose you intend to include the very tiny centrifugal force due to the earth's rotation, and really mean net acceleration."

"Yes, of course," said the Mad Hatter, "if you want to be particular. But your first statement was wrong; weight is not mass, but is gravitational force on a mass."

"Well," said Alice, Jr., "you can tell the muchness, or mass, of anything by weighing it on beam scales against standard chunks of mass that even you call 'weights.' When my mother wants to see if she is getting too fat, she does this and she calls it her weight. I am sure it tells her how much is inside of her, and has nothing to do with the actual value of gravitational force. Besides, a groceryman who lives high up in Mexico City, not far from the equator, or who lives in Canada near sea level, or even grocerymen on the moon, if there are any, will sell the same muchness and sweetness for a pound of sugar, by weighing it against the same 'weights' on a beam scales, and I know the gravitational force that you want to call weight is quite different in these different places. So what you call the weight of a 'weight' doesn't tell you much of anything. But you can tell the mass in pounds just by looking at the numbers stamped on the weights, without the extra bother of thinking about force and acceleration and what you call weight."

"Well," snapped the Mad Hatter, "when you weigh a thing against weights, you can't call the muchness weight; you have to call it mass."

"All right," said Alice, "I won't say weight any more, but will say mass instead. But lots of people don't know that you call gravitational force 'weight,' and they think weight means mass. So you'd better stop using your ambiguous word 'weight,' and say 'gravitational force' instead. But what has that got to do with slugs? I'm always used to measuring mass in pounds, as I've said already."

"No," said the Mad Hatter, "you've got to think about slugs. A pound is just like a slug, but is about $\frac{1}{32}$ of it."

"Now you're getting us all mixed up," said Alice. "Even if we are going to use the word 'mass' instead of 'weight' for muchness, when my grocer has a pound-weight on one side of a beam scales or balance, and some sugar on the other side, and they balance, why can't we call it a

pound of sugar? Then we don't even need to mention weight or slugs or W/g."

"Oh, no," said the Mad Hatter, "you've got to measure muchness in slugs."

"I don't know why," said Alice. "You can't say that W/g in slugs is the only unit for mass. Chemists in England and America measure muchness in pounds, which I am willing to say is the pound-mass instead of the pound-weight. Mechanical engineering handbooks talk about 480 pounds of iron in a cubic foot, and so many pounds of wire per 100 linear feet, and all that. Industry and commerce in the United States and Great Britain really are based on the pound-mass. The chemist's pound-molecule is based on the pound-mass. Whoever heard of a slug-molecule? The international standard of mass isn't a slug; it's the International Prototype Kilogram, and there is a copy of it in the National Bureau of Standards. The standard pound-mass is defined by United States law as a specified fraction of the kilogram, and a representation of this also is kept by the Bureau of Standards.

"Then any pound-mass is what balances a replica of this chunk of muchness called the standard pound-mass, on a beam balance, anywhere, at any altitude, on earth or moon. It has been proved once for all that this gives inertial mass; so we don't have to stop and confuse ourselves, every time we use a beam balance or write an equation, by supposing that we are measuring force. A standard pound-force is how much this pound-mass is pulled at any place where there is the international standard value of gravitational acceleration, 32.1740 ft/sec.[2]

"Everybody was happy when we used to talk about poundals, and then force in poundals was mass in pounds times acceleration, or $F=ma$. But if we want to use the pound-force, which is about 32 times as big as a poundal, we can keep the pound-mass that is used in everyday life, and put a pure number—about 32—in the denominator because the force unit has a numerical value which is about $\frac{1}{32}$ of the value in poundals. We may define this denominator as a pure number, g_c, equal to the absolute value of the International standard value of g, and use it in the equation $F=(m/g_c)a$, without any complications about gravitational forces or slugs. We have exactly the same numbers to work with and the same arithmetic as you do with slugs and the equation $F=(W/g)a$, without the fogginess of this new and high-brow slug, and, as I said before, without having to decide whether W and g are standard or local values and without thought of weight. Frenchmen do the same thing with the kilogram-force and kilogram-mass. Nobody wants to think of a metric slug."

"Well," said the Mad Hatter, "if you are going to work with the aerodynamics people, I keep telling you that you can't measure mass in pounds any more, and have got to measure it as some of them do, as the quotient of gravitational force and gravitational acceleration."

"That sounds like jabberwocky to me," said Alice. "A pound-mass is a pound-mass, whether it is on the moon or anywhere. Do you know the gravitational acceleration on the moon? If you want to compute the forces and accelerations on an airplane at 60,000 ft, you can't say that the gravitational force and acceleration, which your W and g explicitly mean, are practically the same as at sea level."

"But the mass in slugs doesn't change with altitude," said the Mad Hatter, "because the weight and the gravitational acceleration change in the same ratio. And when we use in the equations W and g that explicitly mean actual values, we don't intend that at all, but do intend standard values, W_s and g_s."

"Then it's silly to use W and g," said Alice. "You put in their values that everybody knows are variable, and whose ratio in some units people call slugs, and then you say you don't need the actual values after all. So it's simpler to stick to the pound-mass and the standard pound-force that you know have the same value at all altitudes, instead of using W and g, both of which vary. Then you can use the actual value of gravitational acceleration at high altitudes when it is needed for computation of airplane accelerations.

"All the thermodynamics people that I know talk about a pound of working substance. If you keep on talking about slugs, I suppose next you will want to have me talk about entropy of a slug."

"Why, of course," said the Mad Hatter, "Newton's second law says that force is equal to mass times acceleration, and the only way you can follow Newton is to measure mass in slugs, even with entropy."

"I was told that Newton's second law doesn't say anything of the kind," replied Alice, Jr., "but that he said—in Latin—that force was *proportional* to mass times acceleration. So in Newton's second law I can, as I said, measure force with the standard pound-force and muchness with the

pound-mass, by putting in this proportionality factor with no dimensions, whose numerical value is that of the standard g_s. I still think that's a lot simpler than having to remember that mass is gravitational force divided by g. Why, some of the people who do this are so ashamed of it that they won't even call it a slug, but just say it is a unit of mass. Other people add to the confusion by calling it a gee-pound. But I am going to stick to the good old pound as the unit of mass, just as the grocer does. There was a chunk called the standard pound, in England, maybe before the Pilgrims landed in America, and sealers of Weights and Measures and Bureaus of Standards in both countries have equivalent chunks to this day. It is much simpler to think of mass as something measured by comparison with these standard chunks, than as being the quotient of W and g. And if you do this, you don't have to introduce your concept of weight at all in talking about mass. And as for entropy of a slug, entropy is hard enough to understand, anyway, without having to mix up slugs with it. All the steam tables and ammonia tables give entropy per pound of muchness. Who wants to have all the steam tables done over in slugs? So mechanical and chemical engineers now working on jet propulsion have all of their lives measured the working substance and the values of entropy, enthalpy and heat of combustion, all in terms of the pound-mass as the unit, and so does Professor Keenan's new book on *Thermodynamics of Air*. The plane builders get the mass of the plane in pounds, and the pound-mass and standard pound-force are perfectly good units to keep on using when the thrust on the plane is being computed. The Wright brothers and their successors who started building airplanes were mechanical engineers, and if other people want to come into this territory and make aerodynamic computations, they oughtn't to mix us up by trying to foist slugs on us. Students in colleges have been befuddled for years because machinists, grocerymen, and their professors of thermodynamics and chemistry use the pound-mass, while their professors of theoretical mechanics, who teach them in the next hour, use slugs."

"How does a girl computer like you come to know so much physics?" said the Mad Hatter.

"Well," said Alice, "the talk about slugs and gee-pounds and three different kinds of weight and poundals and kilograms-force and dynes and pressure in bars, was a Wonderland more mixed up than anything the other Alice ever got into, so I studied and found the only way out was to keep on with the good old-fashioned pound-mass."

"Well," said the Mad Hatter, "even if everybody once used the pound-mass and everyday people still use it, lots of people have learned slugs and have come to believe them best, and we have to please them."

"We must re-educate them," said Alice. "Lots of people once believed in caloric and the luminiferous ether and phlogiston and said 'force' where we now say 'energy,' and they re-educated themselves. Mass measured in slugs or as W/g is not essentially wrong like those things, but it has been part of many people's education. So the sluggers must realize that slugs do cause confusion, and that there is a point of view that uses the pound-mass as it is used in everyday life, and that many engineers use it with complete accuracy, with the 32.1740 in Newton's law as a proportionality constant without dimensions. Then the sluggers may be willing to be re-educated.

"Anyway," continued Alice, Jr., "no slugger is going to befuddle me the way they do lots of college students with their slimy old slugs. I am willing to give up the word 'weight' and use the word 'mass' instead, but it's got to be the pound-mass. So nuts to your slugs."

The Mad Hatter didn't seem convinced, but he was so vexed that he had no answer and disappeared. Thoughts of slide rules and notebooks returned to Alice. But she was too confused and sleepy to do more work that night, so she was soon in bed, and, with the soporific melody of the Mock Turtle's song running through her head, fell fast asleep.

NOTHING would do more to promote a rapid advance in human understanding than an annual conference between schoolboys, schoolmasters, and elderly scientists. The scientists should be compelled to attend under pain of losing their pension rights.—LANCELOT HOGBEN, *Mathematics for the millions* (1937), p. 564.

Some Values of Problems and Laboratory for the Nonscience Student

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THE title of this paper presupposes that in the liberal arts college there will be two introductory courses—one for those planning to major in one of the sciences and another for the nonscience students. At the outset it may be pointed out that such an arrangement of courses is not the only alternative. Experience of many years has led some physics teachers to believe that the two courses may be combined satisfactorily into one course, to the advantage of both groups of students; moreover, a single course is more economical of college funds.

The argument for the single course is based on the belief that physics is the same no matter what may be the line of work to which it is applied. *Physics is physics*, whether for the engineer, the medical man, the geologist or the nonscience student. For example, a course in physics for the householder would be one designed to show how physics fits into our everyday household activities. That would be fine if all of our students hoped eventually to be running a home and nothing else. Must we give a special introductory course for each field into which students propose to go for a life's work?

Actually, it is the fundamentals that are important, and the intelligent student will see the applications of the laws and principles of physics to the particular field in which he is interested. An excellent illustration of this result is to be found in a recent letter from a former student who took such a course in general physics without being sure that he would finally take up medicine as a profession. He is now in second-year medical work. The letter reads as follows:

> Some months ago I happened to meet in the cafeteria a college graduate who was applying for admission to the medical school, and he asked me what things in our preparation we would use most in our first year here. I quite surprised him with the amount of physics which one uses in a physiology course, including many things we learned in Physics I which we never believed we would use here. It is not only a few isolated facts and equations, but also the physical approach to a question that is valuable. For instance, in circulation we study velocities, and pressure in various parts of the system; in pulse waves there were velocities and volume flow to be studied and there were plenty of equations relating to these quantities. We even used Bernoulli's principle, which I used to hear so much about.
>
> Electricity has a big place too, of course, especially in muscular activity and in the excitation wave which initiates the heart beat. One experiment, in introduction to electrocardiography, brought up the matter of finding the equipotential lines in a pan of salt solution which conducted a current. Only a few of us had ever heard about such things before! Another time we measured tissue excitability, using condenser discharges as we did in measuring capacitances by comparison.
>
> Mechanics came in more than I expected, especially in muscle action; also subjects like levers, forces and vibratory motion. One example is the relative amount of force and acceleration in the middle and at the ends of an oscillatory movement, such as that of an arm moving back and forth.
>
> There were at least two lectures on the physics of sound and hearing, and more on vision and color. I am not saying anything that is new to you, for you stressed many times during the course the many applications of physics to medicine, but I should like to state my appreciation of a good background in physics during a difficult first year in medical school.

This letter, which came out of the blue, clearly shows that the student who wrote it has an awareness of the relationship between physics and medicine and is trying to bring two fields of science together, with a better understanding of both. Certainly the correlation of first-year physics by this student with his medical studies would warm the cockles of any teacher's heart.

To give one course in physics for engineers and another course in physics for biologists is, after all, like explaining all the details of a joke in the fear that the point of it will not get across. Should we not give our students credit for a modicum of intelligence? This does not mean that in a general course in physics no allusions will be made to the various fields to which the principles of physics may apply. Quite the contrary; the wider the range of applications of the laws and principles of physics, the more intriguing the physics course becomes to the average student. This is why the principle of Bernoulli is so interesting. It has such a wide application. As an illustration in medicine, Boyle's law has many applications. Point them out to your class, but at the same time let it be known that Boyle's law has many applications in chemistry, in meteorology and in kindred fields.

This point of view having been clarified, it becomes obvious that any course in physics,

whether for nonscience students or not, is still, or should be, a course in physics, and should include mathematics and laboratory work, which are of inestimable value in whatever kind of course in physics may be offered to underclassmen in a liberal arts college. To leave all mathematics out of a general course in physics for nonscience students in undergraduate days is, to my way of thinking, depriving a student of one of the greatest contributions that the course can offer. The contribution to which I refer is to show how one can reason about the numerical relations between natural phenomena. The several sections of mathematics are different modes for viewing these relations, and they become a powerful tool even for the nonscience student. Mellor[1] says, "Herbert Spencer has defined a law of nature as a proposition stating that a certain uniformity has been observed in the relations between certain phenomena. In this sense a law of Nature expresses a mathematical relation between the phenomena under consideration." You cannot get away from mathematics no matter what kind of physics course you offer. For example, Boyle's law expresses, within limits, a certain relation between the physical phenomenon of pressure and the phenomenon of volume of a gas under constant temperature. How can one explain such a relationship to a class of nonscience students without (at least indirectly) putting down this relationship in a formula:

$$PV = \text{const.}, \qquad (T \text{ constant}).$$

Without expressing it as a formula may not the student say that $PV^2 = \text{const.}$, or $P^2V = \text{const.}$? In any case, the formula is an equation and, as such, is mathematics. Let us begin to reason about this relationship expressed by a very simple mathematical equation. As the pressure increases, the volume must decrease proportionally if the equation is true. What is going to happen to V when it approaches the value of the sum of the volumes of the molecules forming the gas? Is it not clear how powerful this approach becomes to the problem? Does it not help in actually picturing the process by which a gas is compressed?

It would occur to the writer that a course in physics for nonscience students ought to say something about simple harmonic motion. Every student has seen the periodic vibrations of some mechanism such as a pendulum or a balance wheel in various devices for measuring time. Have you tried to explain simple harmonic or vibratory motion without the use of mathematical formulas and geometric figures? At the outset simple harmonic motion is defined in terms of a geometric figure or an equation. A student, no matter what his inclinations are, should have it demonstrated to him in his college career how valuable a tool mathematics may become.

A nonscience student will not go very far in economics or political science, for example, without encountering a great deal of mathematics and physics. The mathematics of the compound interest law has a very wide application in physics. Where better could an economist get his powers to reason by mathematical deduction than in the applications of mathematics to physics, where he is already quite familiar with the physical facts which he has observed in his everyday life?

We cannot lose sight of the fact that the main divisions of mathematics which are taught in the college curriculum are, after all, the result of the physicist's necessity. Newton had to devise his differential coefficient and a limiting ratio in order to explain an instantaneous velocity. Hamilton remarked as he finished his book on quaternions, "There, thank God, that has no practical value;" but Gibbs salvaged a lot of his work and today vector analysis is an essential part of a physicist's equipment.

Our students while in the army reported back a thorough disgust for the training they had in mathematics. As one writer put it, "their training in mathematics has failed them." Where better can these boys obtain some idea of the power and beauty of mathematics as a tool than in the applications of mathematics to the simple problems of a beginning course in physics?

It was the writer's good fortune to attend recently a meeting of scientists fostered by the Commission on Liberal Education of the Association of American Colleges to discuss science for liberal arts students in a postwar period. In one of our discussions a lively tilt occurred, in which the biologist accused the physicist of not teaching physics so the biology students could apply it to their problems in biology. The physicist, wishing to have a scapegoat on which to lay this sin, called on the mathematician to teach his mathematics so it could be used by physics students. In other words, we find ourselves "passing the buck" by saying that the special students in our department lack the proper training because of what

[1] Mellor, *Higher mathematics for students of chemistry and physics* (Longmans, Green, New York, 1931).

has or has not been given them before getting into our bailiwick. Who better could point out the applications of the laws of physics to biology problems than the well-trained and experienced biologist? Who better can show the applications of mathematics to physics than the mature physicist? This laying the blame for the failure of our students passes from the graduate school to the college, from the college to the secondary school and from there on down through the various grades to the kindergarten.

What we need today is less empty criticism and more constructive contributions in our programs of education. Physics teachers must condition themselves to see more widely how physics fits into the whole scheme of education. The subjects in the college curriculum are not isolated fields of knowledge, but are parts of a whole; as physicists, we should not hesitate to use mathematics, chemistry and other kindred subjects to enliven our own field. When a physics teacher takes up the subject of electromagnetic induction he should have no hesitation in showing how the impact of this discovery has influenced the economic life of our country. On the other hand, how can an economist really understand the significance of the economics of public utilities without an awareness of some of the fundamentals of physics? The economics of primitive man were simple until science and invention got to work. Only as we begin to integrate and coordinate our various fields of knowledge for students will they begin to see life and see it whole. We need the courage of teachers to become fools enough to cross what seem now to be boundaries between various fields of knowledge and try to associate these fields more closely. We try to get our students to do this; why shouldn't we as teachers do the same? To associate mathematics with physics is one of the easiest things we have to do in coordinating subjects.

What of the value of laboratory work for the nonscience student? To take the attitude that laboratory work is not essential for such a student in a liberal arts college is like saying that one can teach a student to play a violin without the use of a violin. How can one fully appreciate the exquisite tones of the violin except he himself can pull from the strings at the tips of his fingers those tones that move mankind? Are not these conceptions rooted and grounded in operational procedures? The same thing holds for laboratory work also. How can one better understand the significance of mass and length than by actually carrying out the operation of measuring mass by an analytic balance, or dynamically, by an inertial balance, or in the case of length by using some such device as a yard stick or a vernier or micrometer caliper to measure it? Teaching science without any laboratory procedure—and this includes demonstrations—is to play Hamlet without Hamlet.

A student who has actually performed a laboratory experiment, let us say the determination of the density of a body, has acquired a discipline in careful and correct observation—in treating his data honestly and dispassionately so as to be able to reason objectively from a given set of conditions to their inevitable consequences. Performing such an experiment and doing it understandingly has started that student to seeing clearly and to thinking straight. This is a discipline that the ancient classics afforded, and it now falls to mathematics and physics to carry on this discipline. All of life is a series of problems and their solutions will be right or wrong. Mathematics and physics afford just the type of reasoning and thinking essential for such work.

I have written dogmatically in the hope that it will start others to thinking and arguing about this important question.

THERE is much to be said for the view that the human intellect is a tool devised for dealing with our material environment and is most reliable when so engaged. Divorced from action on the environment, the human intellect turns in on itself and functions as uselessly as a squirrel in a cage, or as the engine of a car before the clutch has been let in.—Editorial, *Nature*, August 19, 1939.

A Glow Tube Flasher for Demonstrating Condenser Properties

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DURING lectures on the properties of electric circuit elements, the question often arises of how to demonstrate effectively the fact that a charged condenser represents a storage of energy in electric form and that this energy may be converted into other forms of energy. One method of demonstrating this is to charge a condenser of several microfarads capacitance to a potential of a few hundred volts and then short-circuit the terminals. The resulting pistol-like report and accompanying shower of sparks are enough to convince even the most drowsy sophomore at an eight-o'clock lecture that a respectable amount of energy is stored in the charged condenser, and that this energy is available for conversion into light, heat and sound.

This article describes a novel method of demonstrating the energy-storing property of the condenser. The equipment needed consists of two condensers, a neon glow tube and a resistor. By means of this equipment a flashing light may be produced for many minutes, or almost an hour, without the light source being connected to a battery or similar power supply, thus showing how a charged condenser can operate an energy-consuming system for a surprisingly long period of time. The demonstration often impresses the experienced physicist as well as the neophyte.

In Fig. 1, C_1 is a condenser with a capacitance several times that of C_2; R is a resistor of the order of megohms; and T is a neon glow lamp, of the type used in pilot lights, test equipment or exit markers. Let C_1 be given an initial charge corresponding to a few hundred volts potential by means of a d.c. power source, and let the source then be removed. The condenser C_2 will charge up through the resistance R, and discharge C_1 somewhat. When the voltage on C_2 rises to the breakdown voltage E_f of the glow lamp T, the latter will fire, emitting a bright flash of light as the voltage on C_2 falls to the extinction voltage E_e. Then C_2 starts to charge again, discharging C_1 further, until the voltage on C_2 again reaches the breakdown voltage of T, resulting in another flash. This circuit resembles the familiar "sweep circuit," with the charged condenser C_1 replacing the usual steady source of d.c. voltage.

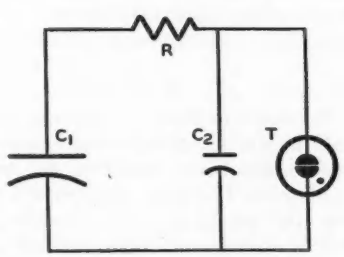

FIG. 1. Glow tube flasher circuit diagram. Condenser C_1 has a capacitance several times that of C_2.

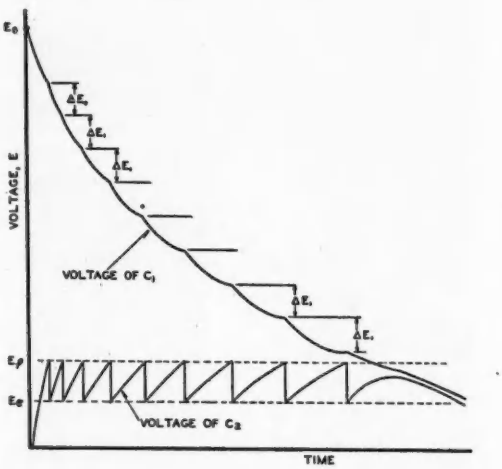

FIG. 2. Time-voltage characteristic of circuit in Fig. 1, showing how the interval between flashes increases as the voltage on C_1 decreases.

The sequence of operation of the circuit is illustrated in Fig. 2. It is apparent from this diagram that the successive lamp flashes occur after continually increasing intervals of time. The neon lamp will continue to flash until C_1 has discharged to such an extent that it can no longer charge C_2 to the firing voltage of T. When this point is reached, the two condensers continue to discharge slowly, at a rate determined by their natural leakage resistances.

The number of flashes to be expected from the system may be calculated with a fair degree of accuracy. The approximate time between the first flash and last flash can be computed in a relatively simple manner, but an exact calcula-

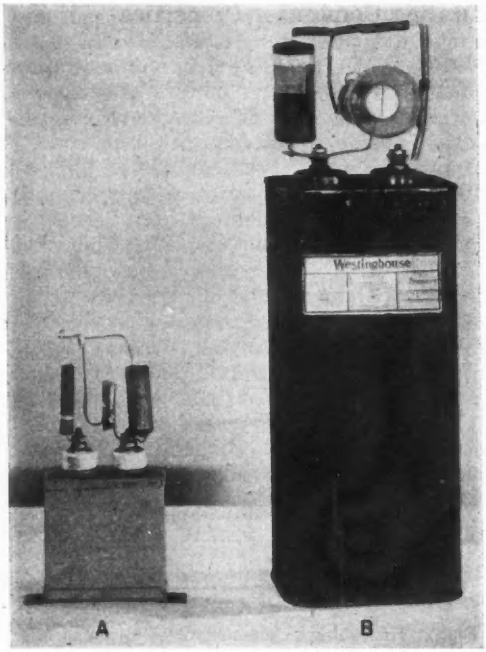

FIG. 3. *A*. This flasher will operate for a period of 11 or 12 min. *B*. Flasher capable of operating for 35 min; while somewhat larger than that shown at *A*, it still can be held easily in one hand.

tion of the total flashing time is a rather laborious process.

To calculate the number of flashes that can be expected, it must be noted that when the voltage of C_2 rises from E_e to E_f (see Fig. 2), then that of C_1 falls by an amount ΔE_1, given by $\Delta E_1 = -(E_f - E_e)C_2/C_1$. This relationship holds for all regions of the curve of Fig. 2, since all the charge on C_2 must come from C_1. It is apparent that if ΔE_1 is small with respect to E_0, then the total number of flashes, N, is almost exactly

$$N = \frac{E_0 - E_f}{|\Delta E_1|} = \frac{E_0 - E_f}{E_f - E_e} \cdot \frac{C_1}{C_2}. \qquad (1)$$

The factors that prevent Eq. (1) from being exact are the fact that the initial flash occurs after C_2 has been charged from zero, not from E_e, and the fact that C_1 does not have to discharge to E_f before it is no longer capable of charging C_2 up to E_f. However, the error in the calculated value of N introduced by these factors is at most two flashes.

To illustrate the surprisingly large number of flashes available from relatively simple, standard laboratory equipment, let C_1 be a 6-μf, 600-v condenser, let C_2 be a $\frac{1}{4}$-μf condenser—both must be high quality paper type, of course—and let T be a neon lamp that fires at approximately 80 v and extinguishes at about 60 v. Then, if C_1 is initially charged to 600 v, we get from Eq. (1),

$$N = \frac{600 - 80}{80 - 60} \times \frac{6}{\frac{1}{4}} = 624 \text{ flashes.}$$

The rate at which the flashes occur is determined by the resistance R, the voltages on the condensers and the capacitances of the condensers. Because of the increasing time interval between flashes, a complicated expression is obtained for the exact time between the first and last flashes. However, since this exact value is of no particular importance, a simple computation of the order of magnitude of this period will suffice. As a first approximation, it can be assumed that the flashes will last for at least as long a period of time as it would take C_1 to discharge from E_0 to E_f through the resistance R if C_2 were short-circuited. Because of the fact that the voltage across C_2 is not zero during the discharge of C_1, but varies between E_e and E_f after the initial flash, C_1 will not discharge as rapidly as the simple calculation will indicate.

However, if the assumption suggested is made, then the total time of flashing will be given by

$$t = RC_1 \ln (E_0/E_f). \qquad (2)$$

If R is 40 megohms, and the other values are the same as those employed in the calculation of total number of flashes, then

$$t = 40 \times 10^6 \times 6 \times 10^{-6} \times \ln (600/80) \leqq 480 \text{ sec.}$$

Thus, with an original voltage of 600 v on C_1, the lamp may be expected to flash for at least 8 min. Experimentally, it is found to flash for 11 or 12 min.

Figure 3, *A*, is a photograph of a model "flasher" having component elements the size of those used in the foregoing computations. The construction has been designed to minimize leakage currents. The glow lamp is a Tattellite pilot light, and produces a glow visible at more than 25 ft in a darkened room. As can be seen, the equipment fits into the palm of the hand. In Fig. 3, *B*, is shown apparatus for a somewhat

more spectacular demonstration. Here C_1 is a 30-μf condenser that can be safely charged to 600 v, C_2 is 1 μf and R is 20 megohms. The lamp is a 2-w, 110-v neon glow lamp, with the resistor removed from the base. The flash produced by this equipment is visible in a lighted auditorium for more than 30 ft.

With the circuit elements of Fig. 3, *B*, the number of flashes will be approximately 780, and the duration of the flashes can be computed to be at least 20 min. Experimentally the flashes are found to last 35 min. If a 60-μf condenser is used for C_1, then the flashes last for more than an hour. Towards the end of the flashing, with the 60-μf condenser, the flashes are about 25 sec apart, whereas the frequency of flashing is about one per second at the beginning.

The apparatus can be demonstrated by having the equipment on the lecture table, with one terminal of C_1 grounded. When the other terminal of C_1 is touched by a lead from a low-impedance d.c. source of the proper voltage, flashing of the light will begin and will last as long as the circuit design permits. This can be as long as a normal lecture period. A more interesting demonstration can be made if the apparatus is charged prior to the beginning of the lecture, and the lecturer then enters the room carrying the lamp that is flashing brightly with no immediately apparent source of energy to operate the light. It is perfectly safe for the lecturer to carry this demonstration equipment when it is fully charged, as long as the large condenser is handled by the case and the terminals are not touched.

A Device for Demonstrating Constancy of Angular Momentum

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IT is well known that the moment of inertia I_p of a cylinder rotating about an axis p through the center and perpendicular to a, the axis of figure, is given by the equation,

$$I_p = M\left(\frac{L^2}{12}+\frac{R^2}{4}\right),$$

where M is the mass of the cylinder, R its radius and L its length. If L is small compared to R, so that the cylinder is a disk, then, $I_p=\frac{1}{4}MR^2$, approximately; for instance, if $L=0.2R$, the approximate expression is correct to within 1.3 percent.

If the disk is rotated about the axis of figure, the moment of inertia I_a is $I_a=\frac{1}{2}MR^2$, or twice the former value. In the first case the rotational energy is $E_p=\frac{1}{8}MR^2\omega_p$, and in the second, $E_a=\frac{1}{4}MR^2\omega_a$, where ω is the angular velocity.

If the disk is mounted so that it can be rotated about the axis p and then suddenly brought to rotate about its axis a, then, by the principle of conservation of angular momentum, $I_p\omega_p=I_a\omega_a$ and, since $I_a/I_p=2$, we have $\omega_p=2\omega_a$; that is, the angular speed must suddenly drop to half its former value. It is evident that if rotational *energy* were conserved, the relation between the angular velocities would be $\omega_p=\sqrt{2}\omega_a$.

To demonstrate in a direct way the facts that angular momentum is conserved and rotational energy is not, an iron disk of radius 19.5 cm and length 1.5 cm has been mounted in gimbals (Fig. 1). The bearings of the outer ring are selected ball bearings mounted in upright supports on a framework. The disk is supported in the ring by pivot bearings. A thin rod running through the ball bearing on one side and along the axis of the ring engages the outer edge of the disk to hold it with its plane in the plane of the outer ring. An endwise pull on the rod releases the disk, whereupon a spring rotates the disk through an angle of 90° to a stop. Thus the disk, initially set in rotation about an axis perpendicular to its own axis, is brought to rotate about its own axis without the application of torque about that axis.

For showing the change in rotational speed, the

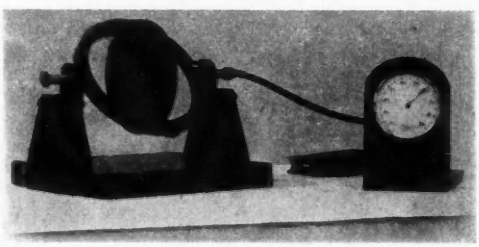

FIG. 1. Photograph of apparatus.

end of the axis of the gimbal ring opposite the rod is connected by a flexible shaft to a speed indicator. For this purpose, a Ford speedometer was altered by adding a section to the torsion spring to increase the deflection.

The demonstration consists in setting the plane of the disk in the plane of the gimbal ring and rotating the whole until the indicator reads about 60. Then the disk is carefully disengaged, and it immediately takes up the position in which it is rotating about its own axis. The speedometer reading drops very nearly to one-half its former value.

The impact of the disk against the stop is always quite noticeable. On this account, and because there is some friction in the bearings, some energy is lost.

The apparatus cannot be used for a precise verification of the constancy of angular momentum, since the moment of inertia of the gimbal ring is not negligible; but observation of the indicator shows that the ratio of angular speeds in the two rotations is nearer 2/1 than it is to $\sqrt{2}/1$.

This device has proved very attractive for class demonstration. It has the advantages that direct readings are given and no calculations are required to show the principle involved.

NOTES AND DISCUSSION

A Graphic Method for the Doppler Effect

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THREE papers on the Doppler effect have appeared recently in the AMERICAN JOURNAL OF PHYSICS.[1] All are based on the concept of wave motion. I should like to call attention to a treatment that makes use of a purely graphical method for determining the change in frequency caused by the motion of the generator, the detector and the reflector.[2] The generator may emit waves or periodic pulses of any kind. The method involves merely the application of the definition of velocity as the slope of a curve in a rectangular system of coordinates representing distance and time, and the use of the fact that when the velocity is constant the slope is constant.

In Fig. 1, generator and detector are moving toward each other along OO_1; ΔT represents the time interval of one pulse, that is, the period. The velocity in space of the generator is v_g; therefore, $v_g\Delta T$ is the distance traveled by the generator during the time of one period. At the beginning of the period, an impulse starts out from O with the velocity c, and its time-distance relation is represented by the line OC. The detector was at O_1 when the generator was at O. Its time-distance relation is represented by the line O_1B; its velocity is v_d. The intersection at E of O_1B and OC determines the time when the first impulse arrives at the detector. Similarly, the last impulse of the period is sent out when the time-distance relation of the generator is given by P on line OA. Since this impulse also has the velocity c, the intersection at G of line AD, which is parallel to OC, with line O_1B denotes its space-time relation at the time of arrival. The vertical distance ΔT_1 between G and E is the time interval during which the detector receives one complete period sent out by the generator in the time interval ΔT. The distance traveled by the detector in the time ΔT_1 is $v_d\Delta T_1$. With the aid of Fig. 2, it is easy to establish the quantitative relation between ΔT and ΔT_1. In Fig. 2, the left-hand side is the same as in Fig. 1, but the triangle EFG (Fig. 1) has been translated parallel to itself

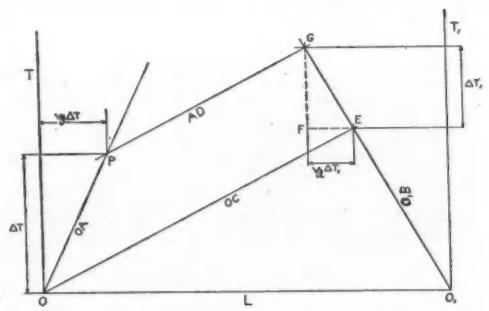

FIG. 1. Time-distance diagram for generator and detector moving towards each other.

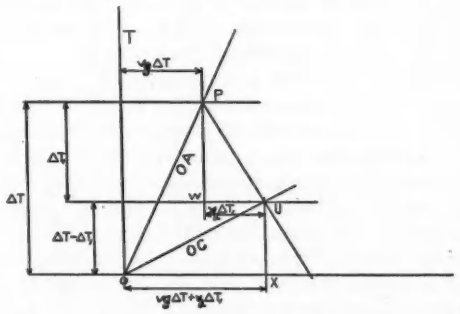

FIG. 2. Time-distance diagram for generator and detector for one period prior to meeting.

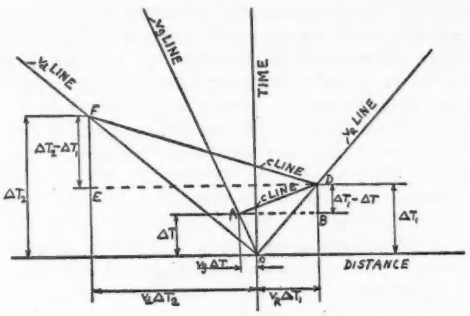

FIG. 3. Time-distance diagram for generator, detector and reflector moving away from one another.

along the lines AD and OC until G coincides with P. From triangle OXU (Fig. 2), we then get the relation $c(\Delta T-\Delta T_1)=v_g\Delta T+v_d\Delta T_1$, or $\Delta T/\Delta T_1=(c+v_d)/(c-v_g)$. But $1/\Delta T$ is n, the frequency of the generator, and $1/\Delta T_1$ is n_1, the frequency of reception by the detector; hence $n_1=n(c+v_d)/(c-v_g)$.

After these explanations, it will not be difficult to see that Fig. 3 gives the expression for the change in frequency when generator, detector and reflector each move with the respective speeds v_g, v_d and v_R. The only further explanations necessary, refer to the double c-lines at point D, which are introduced by the reflection at D. While the velocity is not changed by the reflection, the direction is reversed; this is indicated by the upper c-line, which meets the v_d-line at F. The slopes of the v_g-line and the v_R-line in Fig. 3 are reversed with respect to their equivalents OA and O_1B in Fig. 1, since the signs of the velocities are reversed.

From the triangles ABD and DEF in Fig. 3, we obtain

$$v_g\Delta T+v_R\Delta T_1=c(\Delta T_1-\Delta T),$$
$$v_d\Delta T_2+v_R\Delta T_1=c(\Delta T_2-\Delta T_1);$$

whence

$$(c-v_R)\Delta T_1=(c+v_g)\Delta T,$$
$$(c+v_R)\Delta T_1=(c-v_d)\Delta T_2.$$

Eliminating ΔT_1 by division, and setting $1/\Delta T_2=n_2$ and $1/\Delta T=n$, we obtain $n_2=n(c-v_R)(c-v_d)/(c+v_g)(c+v_R)$. This is the formula given by Perrine[1] for the special case v_g, v_R, v_d all >0.

[1] J. O. Perrine. "Doppler and echo Doppler effect," Am. J. Physics 12, 23 (1944); G. F. Herrenden-Harker, "Doppler effect when both source and observer are in motion," Am. J. Physics 12, 175 (1944); F. E. Fox, "Demonstration of the Doppler effect," Am. J. Physics 12, 228 (1944).
[2] L. Fleischmann. "The extension of the Doppler principle to discontinuous periodic effects," J. Acous. Soc. Am. 15, 103 (1943).

Running Waves from Standing Waves

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INTRODUCTORY textbooks in physics point out that standing waves may be obtained from two trains of running waves. In a more advanced course mention may be made of the converse fact that two trains of standing waves may give rise to running waves. This latter fact may not be of importance physically, but there is satisfaction in knowing that each of these two types of waves may be produced by a suitable combination of waves of the other type. The purpose of this note is to describe a simple model that I have been using for a number of years to illustrate the production of running waves by standing waves.

To illustrate the well-known case of the production of standing waves, there has been on the market a model[1] that fits into a projection lantern. In this model there are three gears, all of the same size and one above another, and three wires, one wire above another, one wire attached to the center of each gear. The general direction of the wires is perpendicular to the plane of the gears, and all are driven by a short crank attached to the middle gear. The top and bottom wires are bent to form helixes, so that their projected shadows are sinusoids. One helix is

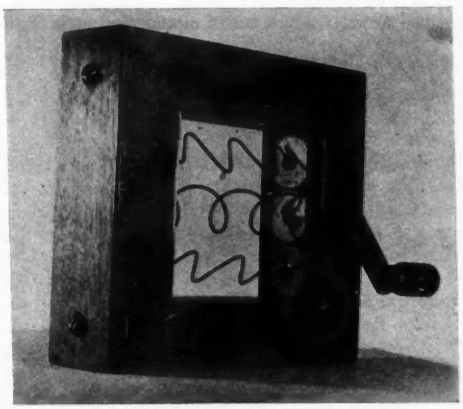

FIG. 1. Model to show production of running waves from standing waves. [Photograph kindly taken by Mrs. H. H. Hatheway.]

right-handed, the other left-handed, and because of the intermediate gear the senses of rotation are the same. Thus the turning of the crank causes the shadows of the top and bottom wires to represent trains of running waves —one train advancing to the right, the other to the left. The middle wire is bent into a sinusoid that lies in one plane, and the turning of this wire represents the standing waves produced by adding the two trains of running waves.

To obtain a similar model showing that standing waves may combine to produce running waves, we recall, for instance, that the sum of $A\cos kx\cos\omega t$ and $A\sin kx\sin\omega t$ is $A\cos(kx-\omega t)$. The model (Fig. 1), like the one just described, is for use in the lantern, contains three gears with attached wires, and is driven by a short crank. The top and bottom wires are flat sinusoids, with the maxima of one 90° from those of the other, and the gears are so set that the planes of these two wires are perpendicular to each other. Thus the shadows of the top and bottom wires

represent trains of standing waves—with the nodes of one halfway between those of the other, and one having its maximum amplitude at the time when the amplitude of the other vanishes. The middle wire is a helix, and is so wound and set that its shadow gives the resultant of the two trains of standing waves. Turning the crank in one direction makes the running waves advance to the right, and turning it in the other direction makes them advance to the left.

[1] This model was devised by Frederick A. Saunders and put out by Gaertner.

Jet-Propulsion Apparatus

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NUMEROUS jet-propulsion devices based on the use of steam, water or compressed air may serve to illustrate Newton's third law of motion, but none of these

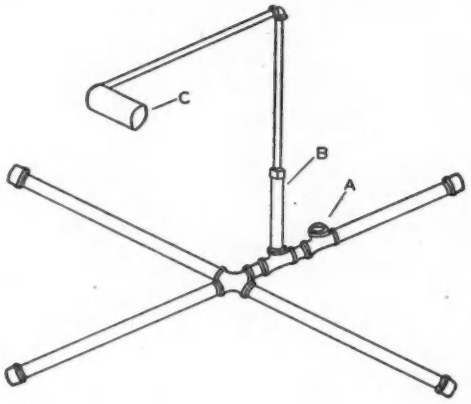

FIG. 1. Assembly of jet-propulsion apparatus: *A*, fuel connection; *B*, bearing assembly; *C*, combustion chamber.

motors actually employs the principle applied in the jet-propelled airplane or rocket, namely, the utilization of expanding combustion gases as the source of energy. The apparatus to be described is operated on the same principle as are the plane and the rocket, and clearly illustrates certain of the interesting features and complex problems associated with combustion jet engines.

Materials required are ¾-, ½- and ¼-in. gas or water pipe and fittings, a small compressor of the paint-spray type and simple machine shop facilities. The apparatus is shown in Fig. 1, the stand being assembled from 2-ft lengths of ¾-in. pipe around a pipe cross. The ends of the pipe are capped and all joints made airtight. The bearing assembly detail is illustrated in Fig. 2, the vertical rotating shaft *F* being made of ¼-in. pipe fitted with a bushing *G* at the lower end to act as a thrust bearing. This pipe is in turn surrounded by a 4-in. piece of ½-in. seamless tubing *D* which serves as a lubricated sleeve between it and the outer ¾-in. upright pipe *C*. A well-oiled packed joint at the top serves as a sealed bearing. The face bushing *E* (Fig. 2) screws into the ¾-in. pipe tee shown at the bottom of *B* (Fig. 1).

The jet is made from a ¼-in., 90° street elbow; the male end is drilled, tapped and plugged with a brass screw cut off even with the end of the elbow. A $\frac{1}{32}$-in. hole is drilled through the plug. The combustion chamber (*C*, Fig. 1) is a size No. 1 can with a hole drilled into the closed end so that the can may be screwed on to the male threads of the elbow. A strip of asbestos paper is wound around the open end of the can so as to project about 1.5 in. beyond the rim to protect the rim from excessive oxidation.

The compressor is modified to supply a mixture of fuel gas and air as follows. The intake louvre is replaced by a wooden plate into which are drilled two holes to accommodate the male threads of gas stopcocks (preferably of large bore). One stopcock is connected to the gas line, the other is left open to the atmosphere. The output line from the compressor is connected by a threaded joint to the apparatus at *A*, Fig. 1.

In operation the compressor is started, the gas turned on, and a flame held to the gas issuing at the mouth of the combustion chamber. This igniting flame must be large, such as is provided by a Bunsen burner or a roll of paper, as a match flame is easily blown out. The composition of the gas mixture is not critical, and the type of flame may be changed by varying the amount of air entering the compressor. Even before the gas is ignited, the rotor will turn slowly, but its velocity will increase after combustion is started.

The chief sources of difficulty are those associated with combustion, friction, gas leakage and weight. The packed joint must be nearly gas-tight and frictionless. An aluminum radio-tube shield substituted for the can reduces

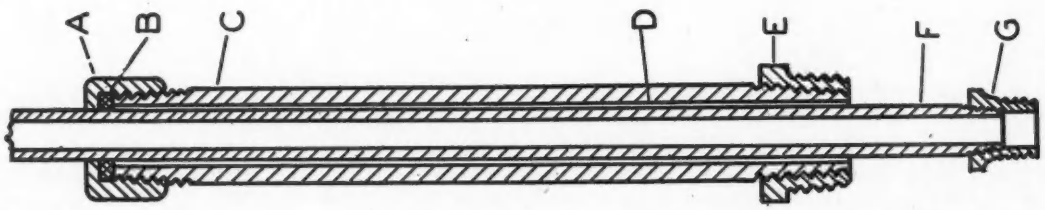

FIG. 2. Bearing assembly of jet-propulsion apparatus: *A*, ¼-in. packing nut; *B*, packing; *C*, ¼-in. pipe; *D*, ½-in. seamless tubing; *E*, ¾×½-in. face bushing; *F*, ¼-in. pipe; *G*, ¼×⅛-in. hex bushing. In the assembled apparatus (Fig. 1) the shaft *F* is, of course, in a vertical position.

weight. If the flame tends to blow itself out, the can should be bent slightly off center with respect to the jet.

The device operates with a loud blast and the emission of a colorless flame which heats the asbestos to redness. The circumferential speed is about 5 ft/sec. If the capacity of the compressor is sufficient, another arm and burner symmetrically placed will increase the velocity.

The Gas Laws Simply Demonstrated

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THE elementary study of a sample of confined air is very effectively extended by means of a device that makes use of familiar ideas. As indicated in Fig. 1, an all-glass stocking-shaped tube contains air in the "toe" region which is confined by mercury in the "heel" and "leg." Tipping the tube for varying the pressure is made convenient by mounting the tube on a board of suitable shape which is attached to the base by a single bolt *A*. This bolt serves as a horizontal axis about which the "stocking" is rotated. An angle of approximately 70° at the "heel" has been found to keep the confined air higher than the confining mercury surface throughout the exercise. This obviates the escape of the air past the mercury. Readings are obtained with such facility that the author has used this laboratory piece in the classroom to obtain data for the class discussion of Boyle's law. For this purpose, room temperature only was used.

By enclosing the "foot" of the "stocking" in a glass cylinder, also shown in Fig. 1, the device is made suitable for the study of Gay-Lussac's law for a number of different volumes. The cylinder contains a mercury thermometer whose scale is used for measuring the volumes of the air sample, in arbitrary units, as well as the temperatures. The 100° mark on the thermometer is placed at the end of the "toe" and marks one end of the sample of air under study. The tubes *T* provide for the inflow and outflow of water at convenient equilibrium temperatures.

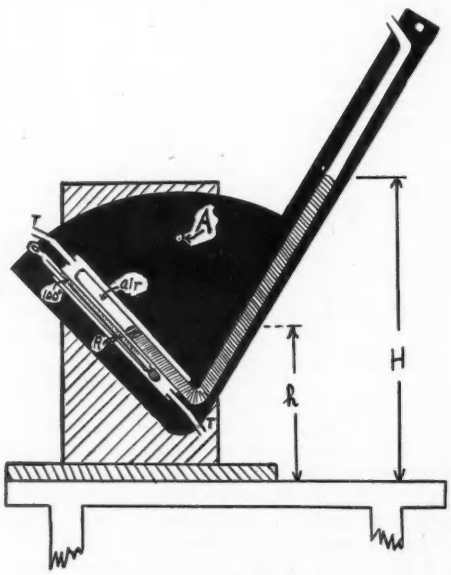

FIG. 1. Apparatus for quickly and easily obtaining data for Gay-Lussac's and Boyle's laws.

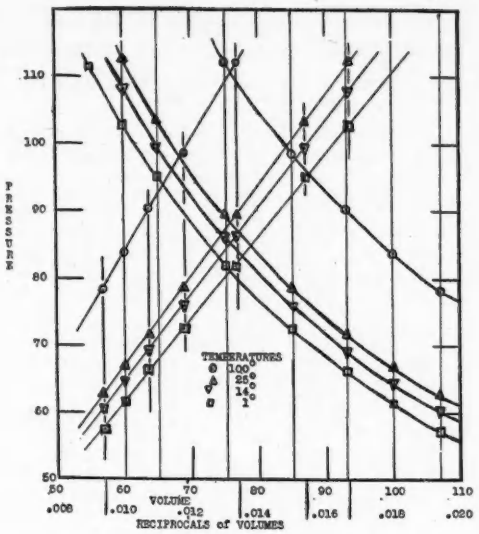

FIG. 2. Pressure-volume curves and pressure-reciprocal volume lines for four temperatures.

Readings are taken as follows. The equipment is initially at room temperature. Rotate the "stocking" until the mercury in the "foot" is opposite some arbitrarily selected mark on the thermometer. If the reading at this mark is *R*, the volume of the air in the uniform tube may be taken as $100-R$. Readings of the heights *H* and *h* of the two

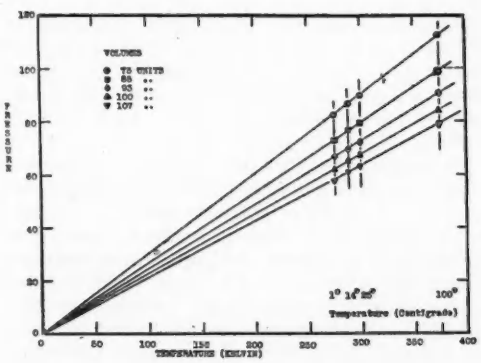

FIG. 3. Pressure-temperature lines for five different volumes. These lines, when extrapolated, meet at the point corresponding to 0°K.

TABLE I. Simultaneously obtained data for Boyle's and Gay-Lussac's laws.

(*a*) Arbitrary values of R, corresponding volumes and reciprocal volumes					
R	25	15	7	0	−7
$V[=100-R]$	75	85	93	100	107
$1/R$	0.0134	0.0118	0.0108	0.0100	0.0094
(*b*) Temperature, 25°C (room temperature)					
H	42.6	36.3	31.8	28.4	25.5
h	29.8	34.1	36.5	38.0	39.2
$H-h$	12.8	2.2	−4.7	−9.6	−13.7
B	76.5	76.5	76.5	76.5	76.5
P	89.3	78.7	71.8	66.9	62.8
PV	6700	6680	6680	6790	6720
(*c*) Temperature, 14°C (water from mains)					
H	41.0	34.9	30.3	27.2	24.3
h	31.3	35.3	37.6	39.0	40.2
P	86.2	76.1	69.2	64.7	60.6
PV	6460	6460	6430	6700	6500
(*d*) Temperature, 1°C (siphoned ice water)					
H	38.6	32.7	28.7	25.4	22.3
h	33.2	36.7	39.0	40.3	41.5
P	81.9	72.5	66.2	61.6	57.3
PV	6140	6150	6150	6160	6130
(*e*) Temperature, 100°C (siphoned boiling water)					
H	54.8	47.6	42.5	38.3	34.5
h	19.2	25.4	28.7	30.9	32.7
P	112.1	98.7	90.3	83.9	78.3
PV	8420	8380	8400	8390	8390

mercury surfaces from the table top are taken by means of a meter stick. If the barometer reading is B, the absolute pressure P is $H-h+B$. The "stocking" is rotated to bring the mercury to each of a series of well-distributed values of R, and the corresponding heights H and h are observed. Such a series is given in Table I, (*a*) and (*b*).

Water at any fixed temperature may now be passed through the cylinder which surrounds the "foot" until an equilibrium temperature is indicated on the thermometer. The "stocking" is then rotated to duplicate the former values of V. When the mercury surface in the "foot" is opposite values of R used in the former series, the corresponding values of H and h are observed. This repetition of the initial volumes is made so that, as Boyle's law data are taken for new temperatures, the data for Gay-Lussac's law are accumulated. Examples of such readings and the results of the calculations are given in Table I, (*c*), (*d*) and (*e*).

The graphs of these data (Fig. 2) show the conventional pressure-volume and pressure-reciprocal volume relationships for each of the temperatures used. They afford a good indication of the effect of changes in temperature. The plotting of the graphs is facilitated by the duplication of the values of V and $1/V$. The range of pressure is best observed from Fig. 2, since only those volumes that could be duplicated for all temperatures were entered in Table I.

The lines of Fig. 3 are representations of the temperature-pressure relations for each of the selected volumes. For the beginning student the repeated indication of the same zero-pressure zero-temperature point is striking evidence of its reality.

Integral Solutions of $x^2+y^2+z^2=t^2$

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IN his note on some Diophantine equations, Lagemann[1] gives an identity from which integral solutions to this equation may be obtained. He mentions that the identity is one of several collected by Dickson.[2]

Although the identity suggested by Lagemann probably gives all of the relatively prime solutions (though not all of them directly), it should be pointed out that most of these identities yield but a small fraction of the relatively prime solutions. As an example, consider the work of Eells,[3] who used a four-parameter identity to obtain 125 relatively prime solutions given by all combinations of the four parameters as they take on values between zero and 15. Only eight of his solutions have values of t less than 100, whereas there are actually 348 such solutions.

TABLE I. Integral solutions of $x^2+y^2+z^2=t^2$.

x	y	z	t	x	y	z	t
1	2	2	3	2	7	26	27
				2	10	25	27
2	3	6	7	2	14	23	27
				10	10	23	27
1	4	8	9	7	14	22	27
4	4	7	9				
				3	16	24	29
2	6	9	11	11	12	24	29
6	6	7	11	12	16	21	29
3	4	12	13	5	6	30	31
				6	14	27	31
2	5	14	15	6	21	22	31
2	10	11	15	14	18	21	31
1	12	12	17	1	8	32	33
8	9	12	17	4	7	32	33
				8	8	31	33
6	6	17	19	7	16	28	33
1	6	18	19	4	17	28	33
6	10	15	19	8	20	25	33
				17	20	20	33
4	5	20	21				
4	8	19	21	6	10	33	35
8	11	16	21	1	18	30	35
4	13	16	21	6	17	30	35
				15	18	26	35
3	6	22	23				
3	14	18	23	3	8	36	37
6	13	18	23	3	24	28	37
				12	21	28	37
9	12	20	25	8	24	27	37
12	15	16	25				

Table I shows the first 50 solutions of this equation, in order of increasing t.

[1] R. T. Lagemann, Am. J. Physics **13**, 268 (1945).
[2] L. E. Dickson, *History of the theory of numbers* (Carnegie Institution of Washington, 1920).
[3] W. C. Eells, Am. Math. Mo. **21**, 269 (1914).

DIGEST OF PERIODICAL LITERATURE

The Renaissance of the "Gaslight" Era

ONCE again gas has become a major factor in modern illumination. In the new gas light of today—the fluorescent lamp—methane has been replaced by mercury and argon, oxidation by electric discharge excitation, and the rare earth oxides of the Welsbach mantle by fluorescent chemicals.

Interest in the gaseous discharge as a source of illumination stemmed from the work of Geissler in about 1855. Even the mythical Captain Nemo had a carbon dioxide discharge lamp to light the submarine Nautilus in *Twenty Thousand Leagues Under the Sea*. This type of lamp was made a reality by D. McFarland Moore in 1913, but he was unable to make it a commercial success.

The first successful discharge lamp, and the only one for many years, was Peter Cooper Hewitt's mercury lamp. Although its efficiency exceeded that of any incandescent lamp of the time, the pronounced bluish-green color made it unsuitable for general use. Numerous attempts were made to overcome this trouble by using combinations of lamps each containing a different gas, but none was very successful. Physicists will understand why. For example, only a very few gases emit visible light efficiently. Moreover, their spectra are discontinuous. Satisfactory color rendition requires that the light source have a continuous spectrum over the visible range and a spectral energy distribution quite similar to that of light from a cloudless sky.

The fluorescent lamp is the nearest approach to this ideal ever devised, and is also the most efficient artificial illuminant yet known. The efficiency is as high as 60 to 65 lu/w, and the lifetime as much as 8000–10,000 hr of continuous burning. Compare these figures with 15 lu/w and 1000 hr for incandescent lamps of comparable size.

Although the fluorescent lamp is simple in construction and in operation, many problems had to be solved to obtain efficient and reliable lamps, and many remain to be solved. The lamp consists of a glass tube with an electrode at each end. The tube is coated inside with a white powder and contains a few milligrams of mercury and a rare gas under a few millimeters of pressure. That is all. But if the lamp is to deliver 60 lu/w, last for at least 5000 hr, have the proper color, and maintain a large fraction of its initial brilliance throughout the lifetime, then its parts must be chosen and assembled in strict accordance with certain complicated physical laws.

It is simple to say that the purpose of the mercury is to furnish vapor that will generate ultraviolet light which is absorbed by the powder, and that the powder re-emits this absorbed energy as visible light. It was less simple to find the conditions under which mercury generates ultraviolet light most efficiently. The resonance line of mercury at 2537A is of most interest, for this is the radiation to which certain types of fluorescent powders respond most strongly. Studies of the efficiency of generation of this particular radiation yield a curve similar to that in Fig. 1.

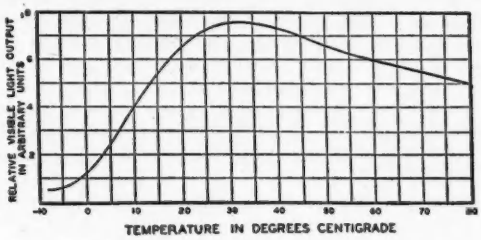

FIG. 1. Light output of a fluorescent lamp *vs.* bulb temperature.

Since the mercury vapor pressure in the lamp is a function of its temperature, the radiation from a mercury arc is a function of the mercury temperature. When the mercury pressure in an arc discharge device is maintained at about 8×10^{-3} mmhg, the 2537A radiation is a maximum and contains 60 percent or more of the total energy of radiation from the arc. Since the only known way to keep the mercury vapor pressure close to 8×10^{-3} mmhg is to have an excess of liquid mercury available and to keep the latter at 38°C, one of the important design factors of a fluorescent lamp becomes apparent. For a desired power dissipation, the bulb must be of such size that the coolest part of the bulb wall will be close to the optimum temperature.

Why is the fluorescent lamp always a long slender tube when a short fat one would so often be desirable? Sound principles demand this slender shape. For example, it has been found that most of the radiation from a discharge originates in the positive column. However, a certain amount of energy is expended at or near the cathode, and it does not contribute materially to the radiation. The fraction of the total energy thus lost increases when the lamp is shortened. On the other hand, the greater the diameter of the lamp the smaller will be the voltage gradient in the positive column and the less efficient the lamp will be, since the losses at the cathode are nearly independent of the tube diameter. Other physical phenomena such as "imprisonment of radiation" must also be understood and considered in lamp design.

The argon in the fluorescent lamp has three important functions: (i) it permits the lamp to be started readily even when the temperature is so low that very little mercury vapor is available to support the discharge; (ii) it enhances the generation of 2537A radiation; and (iii) the inert atoms of argon act as a cushion to protect the cathode from the bombardment of the positive mercury ions. If a lamp contains mercury vapor only, the cathode will be completely ruined in ½ to 20 hr of burning, whereas a lamp that is identical except for a filling of argon under a pressure of 3 mmhg will operate continuously for 8000 to 10,000 hr before the cathode is completely ruined.

Why not increase the pressure of the argon to 6 or even 12 mmhg and thus increase the life still more? At the higher pressures the starting voltage becomes excessive

and the potential gradient in the positive column during operation is so high that over-heating occurs. This lowers the efficiency and may even cause a constricted, or "pipe," discharge. Indeed, for certain low temperature applications it is necessary to reduce the argon pressure to much less than 3 mmhg so that the lamp may be started at all, even though the result is a lamp of much shorter life.

Speaking of "pipe discharges," a serious defect of the early fluorescent lamp was its tendency to "snake;" that is, the discharge became constricted and twirled or whipped back and forth, sometimes with great rapidity. Careful study showed that this phenomenon was due to traces of gaseous impurities of such nature that they could form negative ions. Potential gradients in the discharge are such that these negative ions in effect form a false wall in the tube, thus causing a constricted discharge. Convection currents then cause this constricted column to twirl about. Improved exhaust conditions and high purity of all materials used have practically eliminated this trouble.

The cathode is the heart of all discharge devices. In the fluorescent lamp not only do the usual cathode design problems present themselves, but others as well. For instance, the fluorescent lamp is the only important device of this kind where the same electrode must serve alternately as cathode and as anode. This means that the electrode emits as many electrons during the cathode half cycle as it will receive during the next half cycle. To design a cathode that will operate well under these conditions requires that, in addition to its thermionic emission, electrons must be secured as a result of secondary emission, positive ion bombardment, and the Schottky effect. Careful studies show that not more than 10 percent of the total electron current developed in the discharge is thermionic. During life, the cathode coating material is gradually eroded by the ion bombardment until none remains.

The type of cathode now in widest use consists of a coiled-coil tungsten filament whose primary coiling is packed full of alkaline earth oxides. The diameter and length of the wire, the size of mandrels used in the primary and secondary coiling, and the amount and composition of the alkaline earth coating—all have received long and careful attention.

Problems connected with the fluorescent materials used in these lamps have been a major concern. The chemist has had to develop new analytic procedures as a result of the extraordinary purity requirements. For example, 1 part in 10^6 of iron in certain phosphors will cause a measurable decrease in fluorescence efficiency. The phosphors most widely used are compounded by dry reaction at elevated temperatures, and the intensive study made of the times and temperatures required has resulted in a very rigorous control of all steps in their manufacture.

Since most phosphors depend upon the presence of an activator for their ability to fluoresce, the type and amount of the activator used is of extreme importance. In some phosphors—zinc-beryllium silicate, for example—manganese is the preferred activator, and the proportion of this impurity element determines not only the efficiency of the fluorescence but also its color. Since the color can be varied almost at will, the spectroscopist has been having a field day in determining the factors governing the spectral distribution of various phosphors and in setting up criterions for their control.

The human eye is surprisingly sensitive to variations in color. This lamp's color varies so easily that only by the most careful control can a large number be made that will match for color, not only from lamp to lamp but from month to month and year to year. The lamp parameters mentioned or briefly discussed here have by no means been completely determined. As in so many other new developments, practice has run ahead of theory.—ERWIN F. LOWRY, Sylvania Electric Products, Inc., *Ohio State Univ. Eng. Exp. Station News* **17**, 11–14 (1945).

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